Douglass Houghton Workshop, Section 2, Thu 03/09/17

Worksheet May the Road Rise to Meet You

1. (This problem appeared on a Winter, 2009 Math 116 exam) The quantity

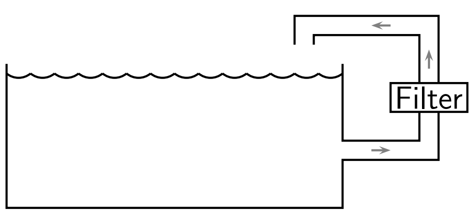
$$\int_1^\infty \frac{dx}{\sqrt{(a^2+x)(b^2+x)(c^2+x)}}$$

roughly models the resistance that football-shaped plankton encounter when falling through water. Note that $a = 1$, $b = 2$, and $c = 3$ are constants that describe the dimensions of the plankton. Find a value of M for which

$$\int_1^M \frac{dx}{\sqrt{(a^2+x)(b^2+x)(c^2+x)}}$$

differs from the original model of resistance by at most 0.001. Hint: make use of the integral and the comparison test.

2. (This problem appeared on a Fall, 2011 Math 116 Exam) An aquarium containing 100 liters of fresh water will be filled with a variety of small fish and aquatic plants. A water filter is installed on the tank to help remove the ammonia produced by the decomposing organic matter generated by plants and fish in the aquarium. The filter takes water from the tank at a rate of 20 liters every hour. The water is then filtered and returned to the aquarium at the same rate of 20 liters every hour. Ninety percent of the ammonia contained in the water that goes through the filter is removed. It is estimated that the fish and plants produce 30 mg of ammonia every hour. Assume the ammonia mixes instantly with the water in the aquarium.



(a) Let $Q(t)$ be the amount in mg of ammonia in the fish tank t hours after the fish were introduced into the aquarium. Find the differential equation satisfied by $Q(t)$. Include its initial condition.

(b) Find the amount of ammonia in the fish tank 3 hours after the fish were introduced into the aquarium. Include units.

(c) What happens to the value of $Q(t)$ in the long run?

3. Last time we guessed that $V_C = R_0\sqrt{\dfrac{g}{R_0+h}}$ is the velocity needed to achieve a circular orbit at height h above the surface of the earth, where

$$\begin{aligned} R_0 &= \text{the radius of the earth } (6371\,\text{km}), \text{ and} \\ g &= \text{the acceleration due to gravity } (9.8\,\text{m/s}^2). \end{aligned}$$

(a) So how long does it take to orbit the earth at height h?
(b) How high would you have to be in order for it to take 24 hours to make one orbit?
(c) What would it be like to orbit at that height around the equator?

4. (From the Fall, 2010 Math 116 final) Let $P(t)$ be the population of birds living on a lake t days after January 1, 2009. It has been noticed that the rate of growth of the population of birds varies depending on the season of the year. To take this into consideration, we assume the rate of growth of the population is equal to $k(t)P$, where $k(t) = \frac{1}{100}\sin(\frac{2\pi}{365}t)$. Hence $P(t)$ satisfies

$$\frac{dP}{dt} = k(t)P.$$

There were 100 birds living on the lake in January 1, 2009.

(a) Solve the differential equation satisfied by $P(t)$ and find the population of birds 100 days after January 1, 2009.
(b) Graph the solution $P(t)$ on your calculator and use this graph to answer the following questions:
   i. What is the maximum and minimum amount of birds living in the lake throughout the year?
   ii. When are the maximum and minimum expected to occur?

5. (Fall, 2014 (the robot chicken semester)) Franklin, your robot, is zipping around the kitchen making his famous "Definitely Not Poison!" soup. His coordinates in the xy-plane are given by the parametric equations

$$x = t^2 - t \qquad\qquad y = -\sin(\pi t)$$

t seconds after he starts making soup. Assume that both x and y are measured in meters.

(a) Calculate dx/dt and dy/dy.
(b) Find all times when Franklin's velocity is zero.
(c) Find Franklin's **speed** when $t = 2$ seconds.
(d) Write an integral that gives the distance traveled by Franklin during his first 5 seconds of zipping around.