Douglass Houghton Workshop, Section 2, Tue 02/21/17

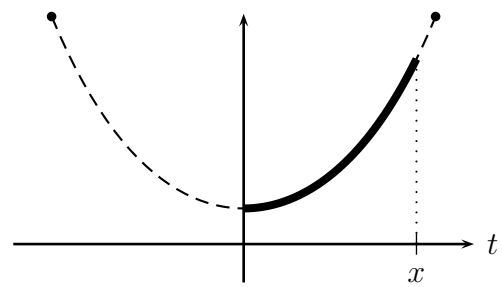
Worksheet Jump Discontinuity

1. If $f(x)$ is any positive-valued function, what is the derivative of $\ln(f(x))$?

2. If $f(x) = \sec(x) + \tan(x)$, what is $f'(x)$?

3. So what is $\dfrac{d}{dx}\ln(\sec(x) + \tan(x))$?

4. We've made some progress finding the shape of a hanging chain. If the shape is given by $F(x)$, then by considering forces and arc length we've shown that
$$F''(x) = \frac{\delta g}{T_0}\sqrt{1 + F'(x)^2}$$
where T_0 is the tension at the bottom of the chain, δ is the mass density of the chain, and g is acceleration due to gravity (all constants). Where to go from here? We'd like to find a formula for $F(x)$.



   (a) Hmmm. No Fs, only F's. And lots of constants. Let $y = F'(x)$, and put all the constants together into one constant. That should make it look better.

   (b) What is y when x is 0? Now you have an initial value to go with your diffeq.

   (c) Separate the variables and solve the differential equation.

5. Last time we found formulas for converting from latitude (ϕ) and longitude (θ) to Cartesian coordinates:
$$\begin{aligned} x &= \cos\phi\cos\theta \\ y &= \cos\phi\sin\theta \\ z &= \sin\phi \end{aligned}$$
Here the origin is the center of the earth, and the radius of the earth is 1. The only catch was that the formulas assume that a point is on the surface of the earth.

   (a) How far is the point $P = (x, y, z)$ from the origin?

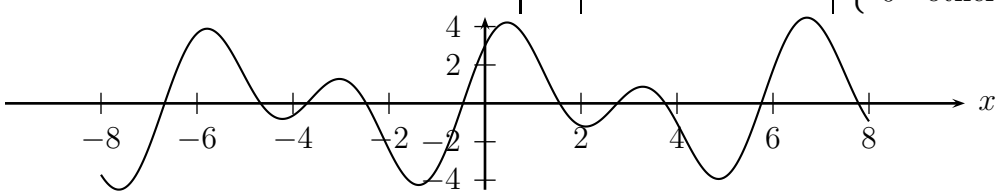
   (b) If you multiply all the coordinates of P by the same number, you get a point that is either directly above or directly below P. Suppose P is below the surface of the earth. What are the Cartesian coordinates of the point on the surface directly above P?

   (c) Find a way to convert the Cartesian coordinates of a point on the surface back to latitude and longitude.

6. There is *still* nothing special at latitude 14°38′53″ N, longitude 78°6′28″ W. It's just a point in the ocean. But, if you were to shoot a neutrino from the middle of the Diag (latitude 42°16′36″ N, longitude 83°44′15″ W) to that point, through the earth's crust, its deepest point would be directly under a very interesting place. Find that place, using Excel.

With a lot of hard work, we filled the table to the right with the values of $\int_{-\pi}^{\pi} f\,dx(x)g(x)\,dx$, where f is the row and g is the column, and m and n are positive integers.

	1	$\sin(nx)$	$\cos(nx)$
1	2π	0	0
$\sin(mx)$	0	$\begin{cases} \pi & \text{if } m=n \\ 0 & \text{otherwise} \end{cases}$	0
$\cos(mx)$	0	0	$\begin{cases} \pi & \text{if } m=n \\ 0 & \text{otherwise} \end{cases}$



7. Suppose $h(x)$ is some function you measure in nature, and its graph looks like the one above. You do some numercal integration and discover that

$$\int_{-\pi}^{\pi} h(x)\,dx = 0 \qquad \int_{-\pi}^{\pi} h(x)\cos(2x)\,dx = 6.28$$
$$\int_{-\pi}^{\pi} h(x)\cos(x)\,dx = 3.14 \qquad \int_{-\pi}^{\pi} h(x)\sin(2x)\,dx = 4.71$$
$$\int_{-\pi}^{\pi} h(x)\sin(x)\,dx = 6.28 \qquad \int_{-\pi}^{\pi} h(x)\cos(nx)\,dx = \int_{-\pi}^{\pi} h(x)\sin(nx)\,dx = 0 \text{ for } n \geq 3.$$

Can you guess a formula for $h(x)$? Use what you know!

8. Find the probability of winning the "Pass" bet in craps.

9. (This problem appeared on the Fall, 2007 Math 116 Final Exam. Really!) Newton's law of cooling (or warming) says that the rate of change of the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding medium. Suppose that a thermometer used by a veterinarian to find the temperature of a sick horse obeys Newton's law of cooling. Further suppose that before insertion the thermometer reads 82° F, after one minute it reads 92°, and after another minute it reads 97° F, and that a sudden convulsion unexpectedly destroys the thermometer after the 97° reading. Call the horse's temperature T_h.

   (a) Write a differential equation for the temperature T (a function of time t) of the thermometer. Your equation may involve the constant T_h.

   (b) Solve the differential equation for T to find a general solution for T. Your solution may include undetermined constants such as T_h.

   (c) Use the temperature data to solve for T.