

Worksheet If I had a Hammer

We've done a lot of integrals with sines and cosines, and filled in this table with the results of $\int_{-\pi}^{\pi} f(x)g(x) dx$:

$f \setminus g$	1	$\sin(nx)$	$\cos(nx)$
1	2π	0	0
$\sin(mx)$	0	π if $m = n$, 0 if $m \neq n$	0
$\cos(mx)$	0	0	π if $m = n$, 0 if $m \neq n$

1. Let $h(x) = 5 + \sin(x) + 2 \cos(x) + 3 \sin(2x) - 5 \cos(2x)$.

(a) Use your calculator to compute:

$$\begin{aligned} \int_{-\pi}^{\pi} h(x) dx &= & \int_{-\pi}^{\pi} h(x) \sin(2x) dx &= \\ \int_{-\pi}^{\pi} h(x) \sin(x) dx &= & \int_{-\pi}^{\pi} h(x) \cos(2x) dx &= \\ \int_{-\pi}^{\pi} h(x) \cos(x) dx &= & &= \end{aligned}$$

(b) Explain the results using the table above.

2. Predict what the integrals in 1a above will be if we change $h(x)$ to

$$h(x) = 2 + 3 \sin(x) - 7 \cos(x) - 4 \sin(2x) + \cos(2x).$$

3. Generalize: What will those integrals be if

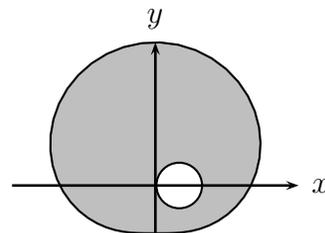
$$h(x) = a_0 + a_1 \cos(x) + a_2 \cos(2x) + b_1 \sin(x) + b_2 \sin(2x).$$

4. (This problem is from a Fall, 2014 Math 116 exam. For some reason, all the exams that term were about robots and chickens.)

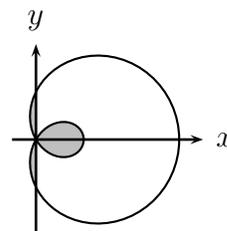
Consider the polar curves

$$r = \cos \theta \quad \text{and} \quad r = \sin \theta + 2.$$

(a) Franklin's robot army occupies the shaded region between these two curves. Find the area occupied by Franklin's robot army.



(b) Your friend, Kazilla, pours her magic potion on the ground. Suddenly, a flock of wild chickens surrounds you. The chickens occupy the shaded region enclosed within the polar curve $r = 1 + 2 \cos \theta$ as shown below. Find the perimeter of the region occupied by the flock of wild chickens.



5. Marlee and Terren have a complicated relationship. Each influences how attracted the other is. Let

x = How attracted Marlee is to Terren

y = How attracted Terren is to Marlee

and suppose that the two are related by the differential equations

$$\frac{dx}{dt} = 2 - x - y \quad \text{and} \quad \frac{dy}{dt} = x - 1.$$

- (a) Use the differential equations to describe the characters in this story. Questions to ask: What kind of a guy is Terren? What strategy can Marlee use to attract him? What happens to Marlee when Terren begins to like her? How should he act to attract her?
- (b) Draw a slope field representing Marlee and Terren's relationship. It should show dy/dx on an xy -plane, for $-4 \leq x, y \leq 4$. What happens in the long run?
- (c) Now write the story of Marlee and Terren's relationship, being true to their characters and the differential equations.
6. There is *still* nothing special at latitude $14^\circ 38' 53''$ N, longitude $78^\circ 6' 28''$ W. It's just a point in the ocean. But, if you were to shoot a neutrino from the middle of the Diag (latitude $42^\circ 16' 36''$ N, longitude $83^\circ 44' 15''$ W) to that point, through the earth's crust, its deepest point would be directly under a very interesting place. Find that place.
7. Find the probability of winning the Hard 4 ($\boxed{. \cdot . \cdot}$) bet in craps.
8. Find the probability of winning the "Pass" bet in craps.
9. Plot the positions of (x, y) given the graphs of $x(t)$ and $y(t)$ below:

