Douglass Houghton Workshop, Section 2, Tue 02/14/17

Worksheet How far that little candle throws its beams! So shines a good deed in a naughty world.

1. We've done a few integrals with sines and cosines. For reasons so far unexplained, we'd like to fill in this table:

	1	$\sin(nx)$	$\cos(nx)$
1			
$\sin(mx)$			
$\cos(mx)$			

with the values of $\int_{-\pi}^{\pi} f(x)g(x)\,dx$, where f is the row and g is the column.

Copy the table on the board, and fill in the entries we know. Then fill in the rest.

2. Consider the "Hard Eight" bet in craps. The bet wins on double fours (⚅⚅) and loses on "soft eight" (⚁⚅ or ⚂⚄) and on 7. If something other than a 7 or 8 is rolled, the bet stays through the next roll.

   (a) Draw the addition table below on the board and fill it in.

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

   (b) Calculate these probabilities:
   - W = the probability of winning on the first roll.
   - L = the probability of losing on the first roll.
   - C = the probability that the game continues to a second roll.

   (c) Calculate the probability of winning on the *second* roll.

   (d) Calculate the probability of winning on the kth roll.

   (e) Calculate the probability of winning on *one of* the first k rolls.

   (f) Calculate the probability of winning the hard-eight bet.

3. Consider the functions "cosh" and "sinh" defined by

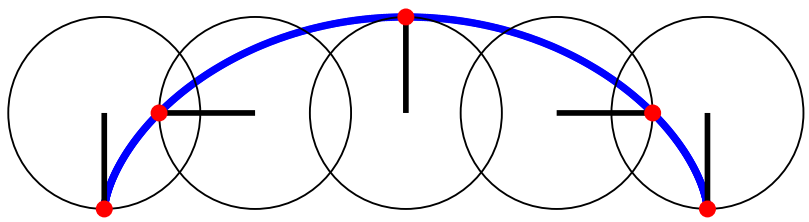
$$\cosh(t) = \frac{e^t + e^{-t}}{2} \qquad \text{and} \qquad \sinh(t) = \frac{e^t - e^{-t}}{2}.$$

   (a) What shape do you get when you plot the parametric curve $x = \cos(t)$, $y = \sin(t)$? Draw it on the board, and write down an equation that relates x to y with no t's.

   (b) What do you get when you plot the parametric curve $x = \cosh(t)$, $y = \sinh(t)$? Plot some points on the board. What happens when t gets big? Try to find an equation relating x and y.

4. There is still nothing special at latitude 14°38′53″ N, longitude 78°6′28″ W. It's just a point in the ocean. But, if you were to shoot a neutrino from the middle of the Diag (latitude 42°16′36″ N, longitude 83°44′15″ W) to that point, through the earth's crust, its deepest point would be directly under a very interesting place. Find that place, and how deep the neutrino is there.

5. Suppose you are watching a frog ride a unicycle in the dark. The only thing you can see is a red reflector which is on the outer edge of the front wheel. As the frog moves from left to right, you see the reflector trace out the path below:



   (a) The wheels in the picture represent snapshots taken every quarter second. The dots are the reflector. Assuming the wheel's center in the first snapshot is $(0, 0)$ and the radius of the wheel is r, fill in the table below with the center's position and the reflector's position at time t.

t	0 sec	1/4 sec	1/2 sec	3/4 sec	1 sec
x_c = center's x	0				
y_c = center's y	0				
x_r = reflector's x					
y_r = reflector's y					

   (b) Find formulas for x_c, y_c, x_r, and y_r in terms of t.

   (c) Find the exact distance traveled by the reflector in one minute. No approximations!

6. (Adapted from a Fall, 2010 Math 116 Exam) In the picture to the right, the graphs of $r = 2$ and $r = 2 - \sin(5\theta)$ are shown.

   (a) Write a definite integral that computes the shaded area.

   (b) Compute the area exactly.

   (c) Write an integral for the length of the boundary of the shaded area.

   (d) Get an approximate answer for that length, using your calculator.