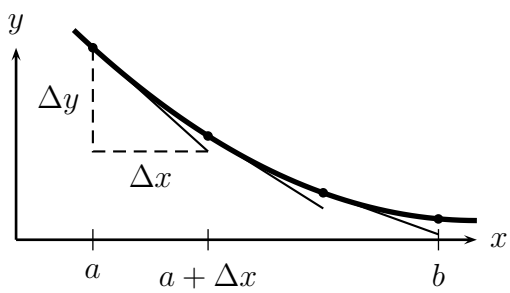
Douglass Houghton Workshop, Section 2, Tue 01/24/17

Worksheet Et tu, Brute?

1. How can we compute the length of a curve $y = f(x)$? Consider cutting it up into small pieces, and approximating each piece with a line segment, as in the picture below.



   (a) How long is the first piece? It is tangent to the curve at a.

   (b) How long is the ith piece?

   (c) Write the left-hand Riemann sum for the length of the curve from a to b.

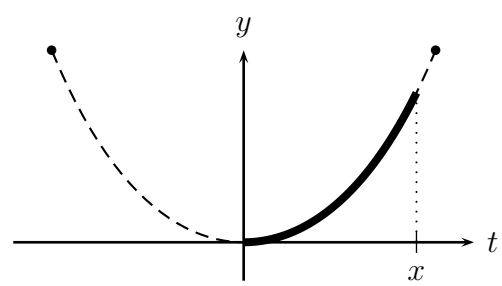
   (d) Now make it into an integral, which will be our formula for arc length.

2. Last time we enumerated the forces on the piece of hanging chain shown here: gravity (mg), leftward tension (T_0) and tension pulling up and to the right (T).

   (a) T can be split into horizontal and vertical components. What are their sizes? (Remember the forces must sum to 0.)

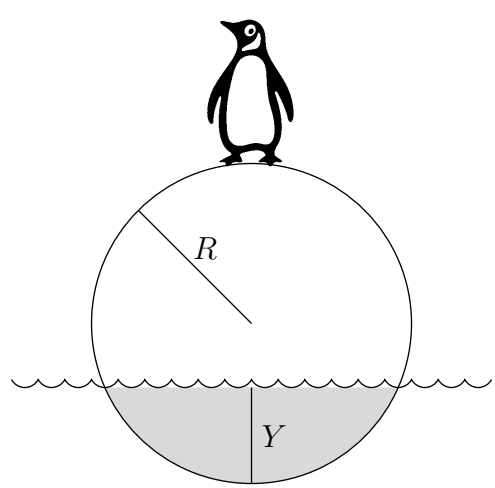
   (b) Let $y = F(t)$ be the shape of the chain. What does the fact that T is pulling in the direction of the chain tell you about the slope of F?

   (c) T_0 is constant as x changes. But the force of gravity is not. Why? How could we find the weight of the chain if we knew $F(t)$?



3. The buoyancy force on a floating object is proportional to the volume of water it displaces. Using this fact, scientists plan to study the weights of penguins by floating beachballs on the ocean and enticing penguins to climb on top. They then measure the depth the ball sinks to, and thereby deduce the penguin's weight.

   So given a beachball of radius R which is partially submerged in the water, find a formula for the volume of the ball which is below the water line when its bottom is at depth Y. Check that your formula makes sense for the values $Y = 0$, $Y = R$, and $Y = 2R$.



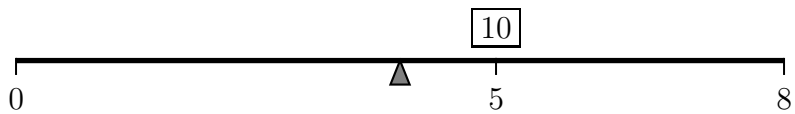
4. Suppose you are pumping water up from a lake to a water tank. The tank is a rectangular solid, with a base that is $46'' \times 38\frac{1}{2}''$, and a height of $38''$. The base is 27 feet above the lake. Water weighs $62.4\,\mathrm{lb/ft^3}$.

   (a) How much work, in ft·lb, will it be to fill the tank?

   (b) It took about 10 oz of gasoline to pump the water up. A gallon of gasoline contains about 132 megajoules of energy, according to Wikipedia. Use the fact that 1 gallon is 128 ounces and 1 ft·lb is 1.355 joules to find the efficiency of the pump.

5. Find $\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\,dx$, given that m and n are positive integers.

6. Imagine an 8-foot long board sitting on a pivot, with a 10 lb bag of flour one foot to the right of center:



The flour will make the board want to rotate clockwise. We say it creates a **torque** on the board. Increasing the weight makes the torque stronger, as does moving the flour to the right. So we say the strength of the torque is

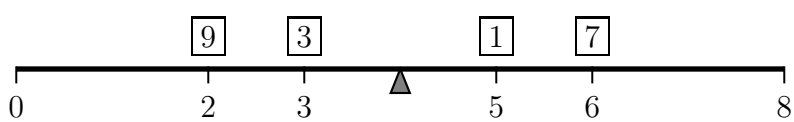
$$T = r \times F$$

where r is the distance from the pivot to the flour, and F is the weight of the flour. So in this case the torque is

$$T = r \times F = (1\text{ft}) \times (10\text{lb}) = 10\text{ft} \cdot \text{lb}.$$

   (a) A weight on the left side of the board will generate a negative (counterclockwise) torque. Where should you place a 3 lb bag of ice to balance out the flour? That's the same as making the total torque equal to 0.

   (b) Whats the total torque of the weights shown here?



   (c) Where should we place the pivot to make the board in part (b) balance?