Douglass Houghton Workshop, Section 1, Mon 03/06/17

Worksheet Jambalaya

1. Find the probability of winning the "Pass" bet in craps.

2. (This problem appeared on the Fall, 2007 Math 116 Final Exam. Really!) Newton's law of cooling (or warming) says that the rate of change of the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding medium. Suppose that a thermometer used by a veterinarian to find the temperature of a sick horse obeys Newton's law of cooling. Further suppose that before insertion the thermometer reads 82° F, after one minute it reads 92°, and after another minute it reads 97° F, and that a sudden convulsion unexpectedly destroys the thermometer after the 97° reading. Call the horse's temperature T_h.

   (a) Write a differential equation for the temperature T (a function of time t) of the thermometer. Your equation may involve the constant T_h.

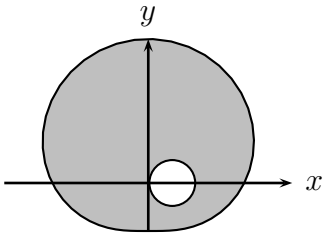
   (b) Solve the differential equation for T to find a general solution for T. Your solution may include undetermined constants such as T_h.

   (c) Use the temperature data to solve for T.

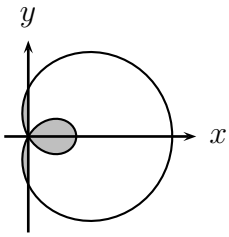
3. (This problem is from a Fall, 2014 Math 116 exam. For some reason, all the exams that term were about robots and chickens.)

   Consider the polar curves

   $$r = \cos\theta \qquad \text{and} \qquad r = \sin\theta + 2.$$



   (a) Franklin's robot army occupies the shaded region between these two curves. Find the area occupied by Franklin's robot army.



   (b) Your friend, Kazilla, pours her magic potion on the ground. Suddenly, a flock of wild chickens surrounds you. The chickens occupy the shaded region enclosed within the polar curve $r = 1 + 2\cos\theta$ as shown below. Find the perimeter of the region occupied by the flock of wild chickens.

4. (Fall, 2016) Consider a compound called Bovinate. The amount $B(t)$ of Bovinate in moles, t hours after an experiment began, satisfies the differential equation

   $$\frac{dB}{dt} = 2B(1-B)(t+B)^2.$$

   (a) Find all equilibria and their stabilities.

   (b) If you start with .5 moles of Bovinate, what will happen in the long run?

5. A spaceship seeks to reach a height y above the surface of the earth. The force of gravity at any time is

$$\boxed{F_g = G\frac{Mm}{r^2}}$$

$G =$ The universal gravitational constant
$M =$ The mass of the earth
$m =$ The mass of the spaceship
$r =$ The distance from the spaceship to the center of the earth.

   (a) How much work will it take to raise the spaceship from the surface of the earth to a point y meters above the surface? Use R for the radius of the earth. Don't assume gravity is constant as the ship moves upward!

   (b) How much work would it take to push the spaceship entriely beyond the reach of Earth's gravity? (Let $y \to \infty$.)

   (c) If the ship is travelling at velocity v, it will have kinetic energy $\frac{1}{2}mv^2$. That energy will be converted into work to move the ship upward. What speed must the ship be going near the surface to leave the earth's gravity well? This is the earth's *escape velocity*.

   (d) Look up the values of G, M, and R, and get a numerical answer in miles per second.

6. Last time we found formulas for converting from latitude (ϕ) and longitude (θ) to Cartesian coordinates:

$$x = \cos\phi\cos\theta$$
$$y = \cos\phi\sin\theta$$
$$z = \sin\phi$$

   Here the origin is the center of the earth, and the radius of the earth is 1. The only catch was that the formulas assume that a point is on the surface of the earth.

   (a) How far is the point $P = (x, y, z)$ from the origin?

   (b) If you multiply all the coordinates of P by the same number, you get a point that is either directly above or directly below P. Suppose P is below the surface of the earth. What are the Cartesian coordinates of the point on the surface directly above P?

   (c) Find a way to convert the Cartesian coordinates of a point on the surface back to latitude and longitude.

7. There is *still* nothing special at latitude $14°38'53''$ N, longitude $78°6'28''$ W. It's just a point in the ocean. But, if you were to shoot a neutrino from the middle of the Diag (latitude $42°16'36''$ N, longitude $83°44'15''$ W) to that point, through the earth's crust, its deepest point would be directly under a very interesting place. Find that place, using Excel.