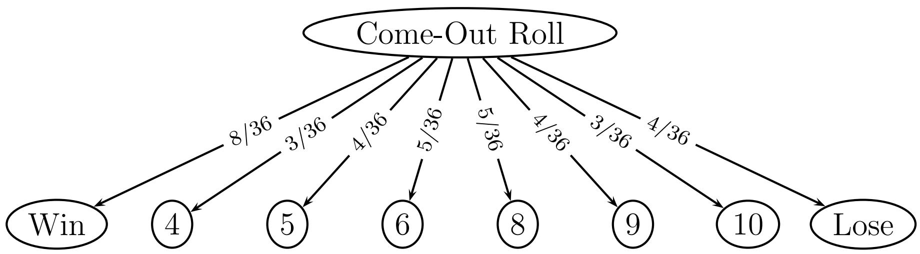
Douglass Houghton Workshop, Section 2, Thu 3/29/2012

Worksheet Quasar

1. We want to find the probability of winning the "Pass" bet in craps. We've analyzed what can happen on the first roll:



   We found that in order to win with a point of 8, we need to roll an 8, then roll some number of non-7's and 8's, then roll an 8.

2. We recently found that $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$ for all x. We did it by taking derivatives and plugging in $x = 0$. Use the same idea to find a series for $\cos(x)$.

3. Consider $f(x) = 1 - \frac{x}{3} + \frac{x^2}{3^2 \cdot 2} - \frac{x^3}{3^3 \cdot 3} + \frac{x^4}{3^4 \cdot 4} - \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{3^n \cdot n}$. The series converges for some values of x; those values constitute the *interval of convergence.*

   (a) Use the ratio test to determine the interval.

   (b) Use other tests to determine what happens at the x-values where the ratio test is inconclusive.

4. On April first, Molli and Natalie like to play practical jokes on people. Their rule, of course, is that every practical joke done on them demands an equal and opposite retaliation. From year to year they keep a mental "balance sheet" that records how much grief they "owe" or are "owed" by their rivals. Due to various interventions by authority, they have been very good the last few years, and they currently "owe" 100 practical jokes.

   They decide that every year, they will pay off 20% of their "debt", by playing pranks on their friends. At the same time, their friends play 5 pranks on them each year.

   (a) What will the "balance" be at the end of 4/1/2012?

   (b) Fill in the table with the balance B_n at the end of $4/1/(2011 + n)$.

n	0	1 (2012)	2 (2013)	3	4	5
B_n	100					

   (c) Find a formula for B_n in terms of n.

   (d) What happens in the long run? (Does the sequence $B_0, B_1, B_2, \ldots$ converge?)

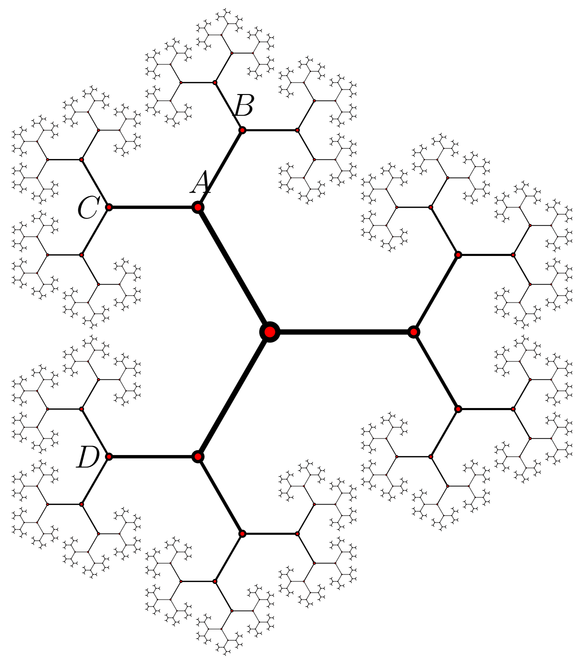
   (e) What if the pranks played happened continuously, instead of once a year. Write a differential equation describing that situation.

5. The figure below contains only 120° angles. As you move away from the center, the line segments get shorter by a factor r. That is, the longest segments (connected to the center) have length 1, the next longest have length r, the next longest after that have length r^2, etc. No '$\ldots$' or '$\sum$' allowed in any of your answers.

Progress: : So far we've found that

- A path from the center to the edge has length
$$1 + r + r^2 + \cdots = \frac{1}{1-r}.$$
- The total height is
$$2\sin(60°)(1 + r + r^2 + \cdots) = \frac{\sqrt{3}}{1-r}.$$
- The distance from the center to the left side is $\frac{1}{2} + r + \frac{1}{2}r^2 + r^3 + \cdots$, which can be split into two series:
$$r + r^3 + r^5 + \cdots = \frac{r}{1-r^2}$$
$$\frac{1}{2} + \frac{1}{2}r^2 + \frac{1}{2}r^4 + \cdots = \frac{1/2}{1-r^2}$$

So the center-to-left distance is $\frac{r+1/2}{1-r^2}$.



(a) Find the distance from the center to the right side of the figure by following a generally northeastward path. Thus find the total width of the figure.

Now of course, the picture could be drawn with any value of r. But if r were too large, the figure would overlap itself, and if r were too small, it would hard to see what's going on. The picture above was drawn by using the largest possible value of r which doesnt cause overlap. Thus the path that goes generally southward from C never crosses the path that goes generally northward from D, but they do approach the same point.

(f) Find the vertical distance from C to D by using a path through the center.

(g) Find the same distance by considering the southward path from C and the northward path from D.

(h) Set them equal and solve for r. Do you recognize this number?

6. (From the Winter, 2010 Math 116 Final Exam) Discover whether the following series converge or diverge, and justify using an appropriate converge test or tests. No credit without justification.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n^2+1}}{n^2-1}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n!(n+1)!}{(2n)!}$ (c) $\displaystyle\sum_{n=2}^{\infty} \frac{\sin(n)}{n^2-3}$