Douglass Houghton Workshop, Section 2, Thu 3/22/2012

Worksheet Once in a Lifetime

1. Find the probability of winning the "Pass" bet in craps.

2. Write out the terms of each series.

   (a) $\sum_{i=0}^{6} i$

   (b) $\sum_{k=3}^{10} (-1)^k k^2$

   (c) $\sum_{\ell=1}^{5} x^{2\ell-1}\ell!$

   (d) $\sum_{m=0}^{\infty} \frac{(2m)!}{m!m!} x^m$

3. Put the series in sigma ($\sum$) notation:

   (a) $9 + 16 + 25 + 36 + \cdots + 100$

   (b) $\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \cdots$

4. Evaluate the following:

   (a) $\sum_{i=1}^{100} 2$

   (b) $\sum_{i=1}^{100} 1/(i+3) - 1/(i+4)$

   (c) $\sum_{i=1}^{\infty} 1/(i(i+1))$ (Hint: $(1/i) - (1/(i+1)) = ?$)

   (d) $1/6 + 1/12 + 1/20 + 1/30 + 1/42 + \cdots$

5. We learned recently that

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

   as long as $|x| < 1$. You can think of both sides of the equation as *functions of* x, and so we have the suprising new idea that a common function we're familar with, $\frac{1}{1-x}$, can be expressed as an infinite series.

   (a) Take the derivative of both sides of the equation above, and find a new such identity.

   (b) Crazy! Maybe other functions we know and like can also be expressed as series? Suppose that:

$$e^x = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

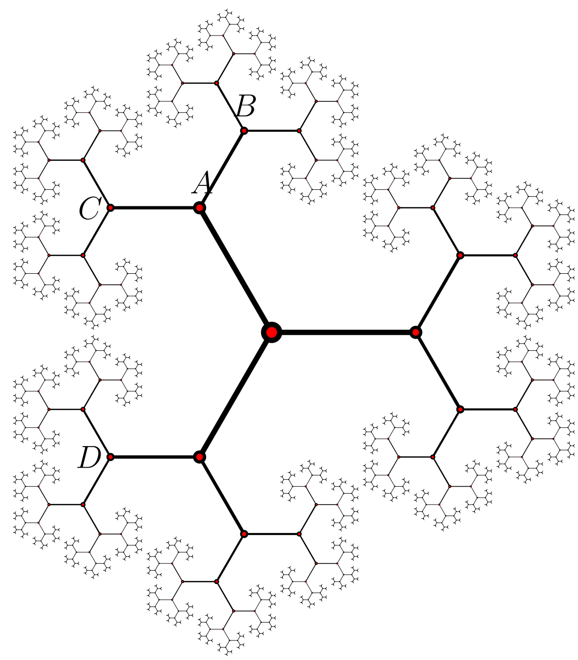
   for some constants $a_0, a_1, a_2, \ldots$, for all x. Take the derivative of both sides, and figure out what the a's are.

6. The figure below contains only 120° angles. As you move away from the center, the line segments get shorter by a factor r. That is, the longest segments (connected to the center) have length 1, the next longest have length r, the next longest after that have length r^2, etc. Assume the branches keep splitting and splitting, ad infnitum. Most of your answers will be in terms of r, but we'll be able to find what r is in part (h). No '...' or '$\sum$' allowed in any of your answers.

   (a) Suppose you start at the center and follow the generally northward path. That is, go to A, then turn right and go to B, then turn left, right, left, etc. How far will you travel after n steps? How far will you travel if you take an infnite number of steps?

   (b) If you take the path described in part (a), how far to the north will you have gone when you reach A? (That is, how much higher on the page is A than the center?) How far north will you have gone when you reach B? When you have gone n steps?

   (c) Use the result of part (b) to give the total height of the figure.



   (d) Find the distance from the center to the left side of the figure by following the generally northwestward path that goes to A, then turns left to C, then right, left, right, etc. This time you want the horizontal distance travelled.

   (e) Find the distance from the center to the right side of the figure by following a generally northeastward path. Thus find the total width of the figure.

Now of course, the picture could be drawn with any value of r. But if r were too large, the figure would overlap itself, and if r were too small, it would hard to see what's going on. The picture above was drawn by using the largest possible value of r which doesnt cause overlap. Thus the path that goes generally southward from C never crosses the path that goes generally northward from D, but they do approach the same point.

   (f) Find the vertical distance from C to D by using a path through the center.

   (g) Find the same distance by considering the southward path from C and the northward path from D.

   (h) Set them equal and solve for r. Do you recognize this number?