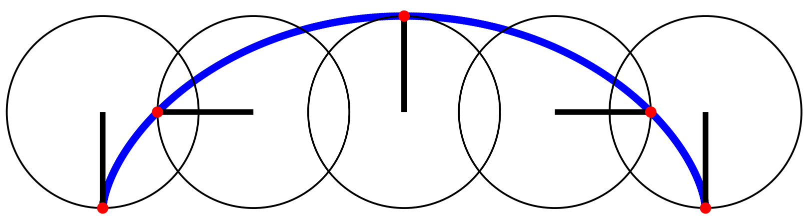
Douglass Houghton Workshop, Section 2, Tue 3/6/2012

Worksheet March Mellifluousness

1. Find the probability of winning the "Pass" bet in craps.

2. Suppose you are watching Shannon ride a unicycle in the dark. The only thing you can see is a red reflector which is on the outer edge of the front wheel. As Shannon moves from left to right, you see the reflector trace out the path below:



   (a) The wheels in the picture represent snapshots taken every quarter second. The dots are the reflector. Assuming the wheel's center in the first snapshot is $(0, 0)$ and the radius of the wheel is r, fill in the table below with the center's position and the reflector's position at time t.

t	0 sec	1/4 sec	1/2 sec	3/4 sec	1 sec
x_c = center's x	0				
y_c = center's y	0				
x_r = reflector's x					
y_r = reflector's y					

   (b) Find formulas for x_c, y_c, x_r, and y_r in terms of t.

   (c) Find the *exact* distance traveled by the reflector in one second. No approximations!

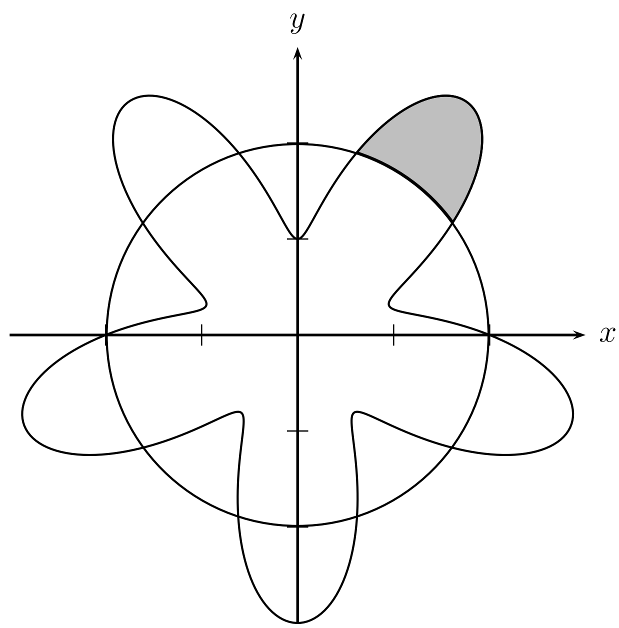
3. (Adapted from a Fall, 2010 Math 116 Exam) In the picture to the right, the graphs of $r = 2$ and $r = 2 - \sin(5\theta)$ are shown.

   (a) Write a definite integral that computes the shaded area.

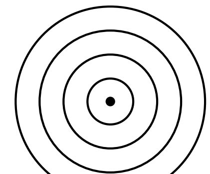
   (b) Compute the area exactly.

   (c) Write an integral for the length of the boundary of the shaded area.

   (d) Get an approximate answer for that length, using your calculator.



4. Consider a game of "continuous darts". The board is circular, as you expect, with radius 1. The goal is to get as close to the middle as possible. If a dart lands a distance r from the bullseye, its score is $1 - r$. (So every number between 0 and 1 is a possible score.)



A novice player throws a dart which lands randomly somewhere on the board. That means that for any region R on the board,

$$\text{Prob(dart lands in } R) = \frac{\text{area of } R}{\text{area of board}}.$$

(a) Fill in the table with the probabilities that the dart scores **below** the given value.

x	0	1/4	1/2	3/4	1
Prob(score $< x$)					

(b) Let x be any number. Find a formula for the probability that the score is less than x.

(c) Find the mean and median scores.

5. (This problem appeared on a Fall, 2007 Math 116 Exam) Consider the integral

$$\int_0^2 \frac{(\sin\sqrt{x}) + 1}{\sqrt{x}}\, dx.$$

(a) Explain why this is an improper integral.

(b) Carefully show, using an appropriate comparison function (that is, without actually evaluating the integral), that this integral converges.

(c) Carefully work out the exact value of the integral.

6. (This problem appeared on a Winter, 2009 Math 116 exam) The quantity

$$\int_1^\infty \frac{dx}{\sqrt{(a^2+x)(b^2+x)(c^2+x)}}$$

roughly models the resistance that football-shaped plankton encounter when falling through water. Note that $a = 1$, $b = 2$, and $c = 3$ are constants that describe the dimensions of the plankton. Find a value of M for which

$$\int_1^M \frac{dx}{\sqrt{(a^2+x)(b^2+x)(c^2+x)}}$$

differs from the original model of resistance by at most 0.001. Hint: make use of the integral and the comparison test.