Douglass Houghton Workshop, Section 2, Thu 2/23/2012

Worksheet Lumberjacks are OK

Recall out recent results:

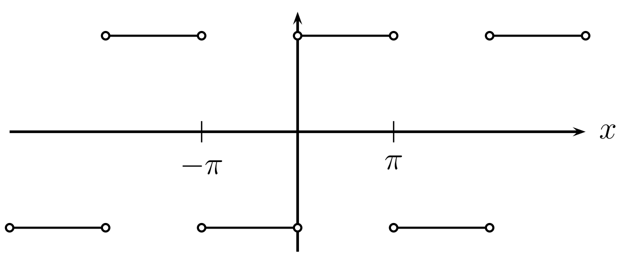
	1	$\sin(nx)$	$\cos(nx)$
1	2π	0	0
$\sin(mx)$	0	$\begin{cases} \pi & \text{if } m=n \\ 0 & \text{otherwise} \end{cases}$	0
$\cos(mx)$	0	0	$\begin{cases} \pi & \text{if } m=n \\ 0 & \text{otherwise} \end{cases}$

So it follows that if

$$\begin{aligned} f(x) &= a_0 + a_1\cos(x) + a_2\cos(2x) + \cdots \\ &\quad + b_1\sin(x) + b_2\sin(2x) + \cdots \\ \text{then } a_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx \\ \text{and } b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx. \end{aligned}$$

1. Consider the square wave:

$$f(x) = \begin{cases} -1 & \text{if } -\pi < x < 0 \\ 1 & \text{if } 0 < x < \pi \end{cases}$$



and that pattern is repeated every 2π.

Suppose the square wave can be written in terms of sines and cosines, as in the top right corner of the page. Find the a_n and the b_n.

2. We know that $V_C = R_0\sqrt{\dfrac{g}{R_0+h}}$ is the velocity needed to achieve a circular orbit at height h above the surface of the earth, where

$$\begin{aligned} R_0 &= \text{the radius of the earth } (6371\,\text{km}), \text{ and} \\ g &= \text{the acceleration due to gravity } (9.8\,\text{m/s}^2). \end{aligned}$$

On Monday we found that the time it takes to make such an orbit is

$$2\pi(R_0+h)/V_C = \frac{2\pi(R_0+h)}{R_0\sqrt{\frac{g}{R_0+h}}} = \frac{2\pi}{R_0\sqrt{g}}(R_0+h)^{3/2}.$$

How high must you be for that to equal 24 hours?

Remember what we found when computing the probabilitiy of the "Hard Eight" bet in craps:

$$\text{Prob of winning in } n \text{ rolls} = W + WC + WC^2 + \cdots + WC^{n-1} = W\frac{1-C^n}{1-C}$$

$$\text{Prob of winning} = W + WC + WC^2 + WC^3 + \cdots = \frac{W}{1-C}.$$

3. Find the probability of winning the "Pass" bet in craps.