Douglass Houghton Workshop, Section 2, Thu 2/16/2012

Worksheet Jump for Joy

With a lot of hard work, we have filled in the table to the right with the values of $\int_{-\pi}^{\pi} f(x)g(x)\,dx$, where f is the row and g is the column.

	1	$\sin(nx)$	$\cos(nx)$
1	2π	0	0
$\sin(mx)$	0	$\begin{cases} \pi & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}$	0
$\cos(mx)$	0	0	$\begin{cases} \pi & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}$

1. Let $h(x) = 5 + \sin(x) + 2\cos(x) + 3\sin(2x) - 5\cos(2x)$.

   (a) Use your calculator to compute:

   $$\int_{-\pi}^{\pi} h(x)\,dx = \qquad \int_{-\pi}^{\pi} h(x)\sin(2x)\,dx =$$
   $$\int_{-\pi}^{\pi} h(x)\sin(x)\,dx = \qquad \int_{-\pi}^{\pi} h(x)\cos(2x)\,dx =$$
   $$\int_{-\pi}^{\pi} h(x)\cos(x)\,dx =$$

   (b) Explain the results using the table above.

2. The Mercury memorial has this equation on it:

   $$V_C = R_0\sqrt{\frac{g}{R_0 + h}}.$$

   What does it mean?

3. Find the probability of winning the "Hard Eight" () bet in craps.

   Progress: We have shown that the probability of winning on the ith roll is $\left(\frac{1}{36}\right)\left(\frac{25}{36}\right)^{i-1}$, and we know we need to add those up to get the total probability.

   (a) Let $W = \frac{1}{36}$ and $C = \frac{25}{36}$. So the probability of winning within the first n rolls is

   $$P_n = W + WC + WC^2 + WC^3 + \cdots + WC^{n-1}.$$

   Copy that on the board, and under it write the sum for $P_n \cdot C$.

   (b) So what is $P_n - P_nC$?

   (c) Now find an expression for P_n that doesn't contain "$\sum$" or "$\cdots$".

   (d) Take the limit as $n \to \infty$ of P_n to get the exact probability of winning the Hard Eight bet in *any* number of rolls.

4. Consider an ecosystem with two species, roadrunners and coyotes. Their populations are represented by x and y, respectively, and they are changing according to these differential equations:

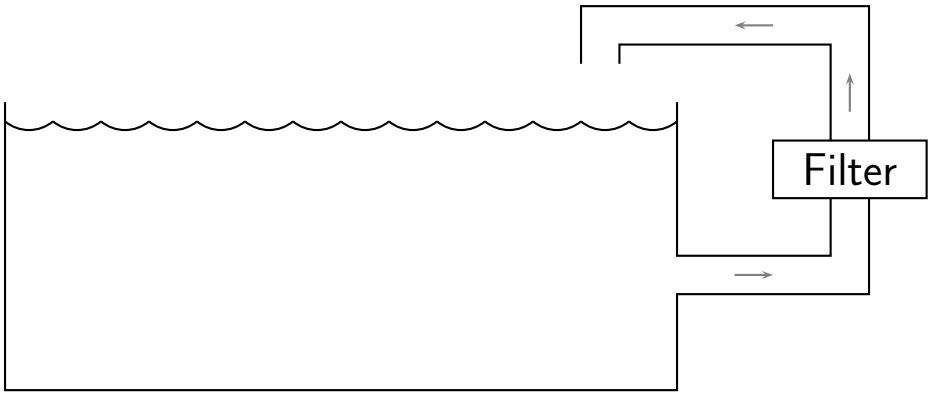
$$\frac{dx}{dt} = 0.2x - 0.005xy \qquad \text{and} \qquad \frac{dy}{dt} = -0.5y + 0.01xy.$$

   (a) Think of some rationale to explain the differential equations.

   (b) Draw a slope field for the system, for $0 \leq x \leq 100$, $0 \leq y \leq 100$. Describe the solution curves.

   (c) Can you use separation of variables to find equations for the solution curves?

5. (This problem appeared on a Fall, 2011 Math 116 Exam) An aquarium containing 100 liters of fresh water will be filled with a variety of small fish and aquatic plants. A water filter is installed on the tank to help remove the ammonia produced by the decomposing organic matter generated by plants and fish in the aquarium. The filter takes water from the tank at a rate of 20 liters every hour. The water is then filtered and returned to the aquarium at the same rate of 20 liters every hour. Ninety percent of the ammonia contained in the water that goes through the filter is removed. It is estimated that the fish and plants produce 30 mg of ammonia every hour. Assume the ammonia mixes instantly with the water in the aquarium.



   (a) Let $Q(t)$ be the amount in mg of ammonia in the fish tank t hours after the fish were introduced into the aquarium. Find the differential equation satisfied by $Q(t)$. Include its initial condition.

   (b) Find the amount of ammonia in the fish tank 3 hours after the fish were introduced into the aquarium. Include units.

   (c) What happens to the value of $Q(t)$ in the long run?