Douglass Houghton Workshop, Section 2, Tue 2/14/2011

Worksheet If music be the food of love, play on

1. Find the probability of winning the "Don't Pass" bet in craps.

   Progress: We have shown that the probability of winning on the nth roll is $\left(\frac{25}{36}\right)^{n-1}\left(\frac{1}{36}\right)$.

2. There is *still* nothing special at latitude 14°38′53″ N, longitude 78°6′28″ W. It's just a point in the ocean. But, if you were to shoot a neutrino from the middle of the Diag (latitude 42°16′36″ N, longitude 83°44′15″ W) to that point, through the earth's crust, its deepest point would be directly under a very interesting place. Find that place, and how deep the neutrino is there.

   Progress: Last time we found formulas for converting from latitude and longitude to Cartesian coordinates:

   $$x = r\cos\phi\cos\theta$$
   $$y = r\cos\phi\sin\theta$$
   $$z = r\sin\phi$$

   where ϕ is latitude, θ is longitude, and r is the distance from the point in question to the center of the earth (equal to about 6371 km for points on the surface).

3. Marisa and Bob have a complicated relationship. Each influences how attracted the other is. Let

   $$x = \text{How attracted Marisa is to Bob}$$
   $$y = \text{How attracted Bob is to Marisa}$$

   and suppose that the two are related by the differential equations

   $$\frac{dx}{dt} = 2 - x - y \qquad \text{and} \qquad \frac{dy}{dt} = x - 1.$$

   (a) Make up a story which explains the differential equations.

   (b) Draw a slope field representing Marisa and Bob's relationship. It should show dy/dx on an xy-plane, for $-4 \le x, y \le 4$. What happens in the long run?

4. Recall the "Field" bet in craps.

   (a) Suppose we bet a dollar on Field, for 36 rolls in a row. In an average run, how many bets will we lose? How many will we win even money? How many will we win double?

   (b) How much money does the casino make, on average, on 36 bets?

5. We've done a few integrals with sines and cosines. For reasons so far unexplained, we'd like to fill in this table:

	1	$\sin(nx)$	$\cos(nx)$
1			
$\sin(mx)$			
$\cos(mx)$			

with the values of $\int_{-\pi}^{\pi} f(x)g(x)\,dx$, where f is the row and g is the column.

(a) Copy the table on the board.

(b) Fill in the entries for $\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\,dx$ and $\int_{-\pi}^{\pi} \sin(mx)\sin(nx)\,dx$, which we calculated before the exam. Remember there were two cases for the second one.

(c) Fill in the top row and left column.

(d) How about $\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\,dx$? Hint: Remember the trick we used for $\sin(mx)\sin(nx)$.

6. (This problem appeared on the Fall, 2007 Math 116 Final Exam. Really!) Newton's law of cooling (or warming) says that the rate of change of the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding medium. Suppose that a thermometer used by a veterinarian to find the temperature of a sick horse obeys Newton's law of cooling. Further suppose that before insertion the thermometer reads 82° F, after one minute it reads 92°, and after another minute it reads 97° F, and that a sudden convulsion unexpectedly destroys the thermometer after the 97° reading. Call the horse's temperature T_h.

(a) Write a differential equation for the temperature T (a function of time t) of the thermometer. Your equation may involve the constant T_h.

(b) Solve the differential equation for T to find a general solution for T. Your solution may include undetermined constants such as T_h.

(c) Use the temperature data to solve for T.