Douglass Houghton Workshop, Section 2, Thu 2/9/2012

Worksheet Hard Eight

1. Consider the "Hard Eight" bet in craps. The bet wins on double fours (⚃⚃) and loses on "soft eight" (⚁⚅ or ⚂⚄) and on 7. If something other than a 7 or 8 is rolled, the bet stays through the next roll.

   (a) Draw the addition table below on the board and fill it in.

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

   (b) Calculate these probabilities:
   - W = the probability of winning on the first roll.
   - L = the probability of losing on the first roll.
   - C = the probability that the game continues to a second roll.

   (c) Calculate the probability of winning the hard-eight bet.

2. There is *still* nothing special at latitude $14°38'53''$ N, longitude $78°6'28''$ W. It's just a point in the ocean. But, if you were to shoot a neutrino from the middle of the Diag (latitude $42°16'36''$ N, longitude $83°44'15''$ W) to that point, through the earth's crust, its deepest point would be directly under a very interesting place. Find that place, and how deep the neutrino is there.

   Before the exam we worked on converting latitude ϕ, longitude θ, radius r to cartesian coordinates. We got as far as $z = r\sin\phi$. Define the x- and y-axes, and find a way to compute x and y from ϕ and θ. Hint: Look down on the earth from above the North Pole.

3. We've done a few integrals with sines and cosines. For reasons so far unexplained, we'd like to fill in this table:

	1	$\sin(nx)$	$\cos(nx)$
1			
$\sin(mx)$			
$\cos(mx)$			

   with the values of $\int_{-\pi}^{\pi} f(x)g(x)\,dx$, where f is the row and g is the column.

   (a) Copy the table on the board.

   (b) Fill in the entries for $\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\,dx$ and $\int_{-\pi}^{\pi} \sin(mx)\sin(nx)\,dx$, which we calculated before the exam. Remember there were two cases for the second one.

   (c) Fill in the top row and left column.

   (d) How about $\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\,dx$? Hint: Remember the trick we used for $\sin(mx)\sin(nx)$.

4. Suppose you want to calculate

$$I = \int \frac{2x+6}{(x-3)(x-7)}\,dx.$$

   (a) Hmm, that looks hard. But what if you had something like this:

$$\int \left(\frac{A}{x-3} + \frac{B}{x-7} \right) dx$$

   where A and B are constants. You could find that, right? Do it.

   (b) That wasn't so bad. And it's related to the initial problem. What do you get if you add the two fractions in part (a), by finding a common denominator?

   (c) Now, what do A and B have to be to make the answer to part (b) equal the integrand of I?

   (d) Now find I.

   (e) When does this trick work?

5. Imagine a population P of ducks on an island. P changes over time. More ducks means the ducks reproduce faster, until the population gets too big, and then overcrowding makes reproduction slow. Eventually, if the population passes 500, then ducks begin to die off faster than they are born.

   Model this system with a differential equation of the form

$$\frac{dP}{dt} = f(P).$$

   That is, come up with an f which has the properties outlined in the story.