Douglass Houghton Workshop, Section 2, Thu 1/26/2012

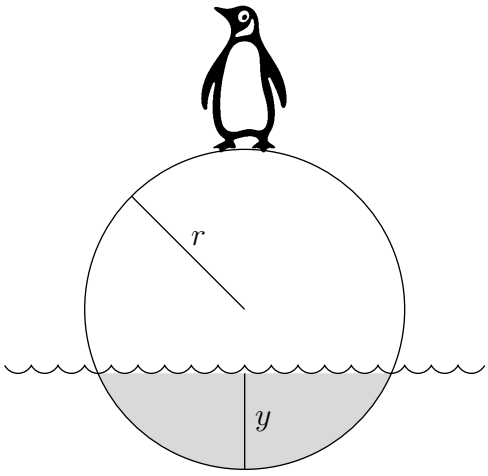
Worksheet Great Balls of Fire

1. There is *still* nothing special at latitude 14°38′53″ N, longitude 78°6′28″ W. It's just a point in the ocean. But, if you were to shoot a neutrino from the middle of the Diag (latitude 42°16′36″ N, longitude 83°44′15″ W) to that point, through the earth's crust, its deepest point would be directly under a very interesting place. Find that place, and how deep the neutrino is there.

   Last week we worked on converting latitude ϕ, longitude θ, radius r to cartesian coordinates. We got as far as $z = r\sin\phi$. Define the x- and y-axes, and find a way to compute x and y from ϕ and θ. Hint: Look down on the earth from above the North Pole.

2. The buoyancy force on a floating object is proportional to the volume of water it displaces. Using this fact, a group of scientists plans to study the weights of penguins by floating beachballs on the ocean and enticing penguins to climb on top. They then measure the depth the ball sinks to, and thereby deduce the penguin's weight.

   So given a beachball of radius r which is partially submerged in the water, find a formula for the volume of the ball which is below the water line when its bottom is at depth y. Check that your formula makes sense for the values $y = 0$, $y = r$, and $y = 2r$.



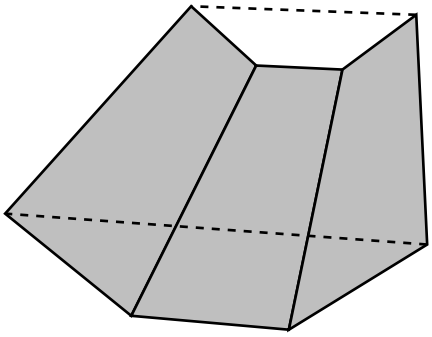
3. (This problem appeared on a fall, 2011 Math 116 exam) In order to fuel a late-night study session at the UGLi, you pull out a can of Bolt Kola, a highly caffeinated soft drink. This particular brand of pop comes in a cylindrical, aluminum can with a removable top. You want to know how much force is exerted on the sides of the can by the drink. You pull out your trusty ruler and find the base of the can has a radius of 3.5 centimeters (0.035 meters) and a height of 16 centimeters (0.16 meters). A quick Google search informs you that the density of Bolt Kola is $1030\,\mathrm{kg/m^3}$.

   (a) Calculate the force exerted by the drink on the bottom of the can.

   (b) Write an expression giving the force exerted by the drink on a slice of the cylindrical wall of the can h meters above the base and of thickness Δh.

   (c) Calculate the total force exerted by the drink on the sides of the can (with the top removed). Show all work to receive full credit.

4. The picture to the right shows a section of the Los Angeles river, whose sides are lined with concrete. It is currently full of water, but you need to empty it so you can film a car chase scene for a movie (as in *Terminator 2*, *Grease*, *Gone in 60 Seconds*, *Buckaroo Banzai*, etc.) It is 100 meters long, 17 meters deep, 40 meters wide at the top and 20 meters wide at the bottom. Find the work required to pump all the water up to the top of the river.



5. (This problem appeared on a Winter, 2003 Math 116 Exam) A decorative table leg (see diagram) is manufactured so that it is the volume of rotation of the function $f(x) = 4 + \sin(x)$ between $x = 0$ and $x = 16\pi$.

   (a) What is the volume of the table leg? Show all work, but you may evaluate any integrals on your calculator.

   (b) Heres a plausible shortcut: Replace the complicated shape with a cylinder with height 16π and radius 4 (because the average radius above is 4), and apply the volume formula for a cylinder. Is the shortcut valid? Explain briefly.