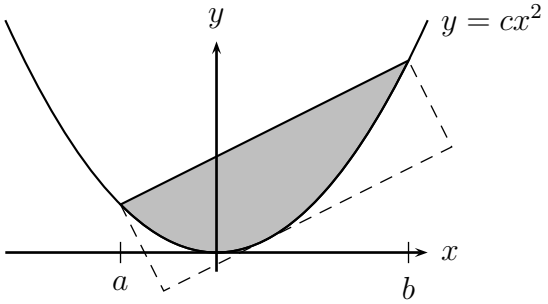
Douglass Houghton Workshop, Section 2, Thu 1/5/12

Worksheet Archimedes

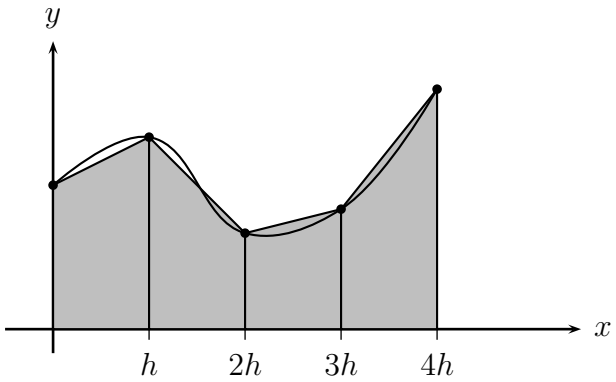
1. Find the volume of a plastic cup, using any method you like. What assumptions do you need to make? What measurements?

2. (a) Find the area of the shaded region in the picture to the right. Make the answer as simple as possible.

   (b) Find the area of the dashed rectangle, which is tangent to the curve.



3. Last fall we talked a lot about left-hand and right-hand sums for approximating integrals. You may have thought to yourself: "Rectangles seem like a very poor approximation for the area under a curve. There's all that wasted space at the top. Can't we do better?" The truth is that rectangles are the simplest way to approximate a definite integral, but we *can* do better. The simplest improvement would be to use trapezoids instead of rectangles:



Suppose we have $f(0) = y_0, f(h) = y_1, \ldots, f(4h) = y_4$.

   (a) Write a formula in terms of the y's for the left-hand sum approximation of $\int_0^{4h} f(x)dx$ with 4 rectangles.

   (b) Write a similar formula for the right-hand sum approximation of $\int_0^{4h} f(x)dx$.

   (c) Find the area of the leftmost trapezoid in terms of h, y_0, and y_1.

   (d) Write a formula for the trapezoid approximation of $\int_0^{4h} f(x)dx$, with 4 trapezoids. How is it related to the formulas you found in (a) and (b)?

   (e) How can you do better than trapezoids? Hint: First we approximated the curve with constant functions, then with line segments. What could we try next?

4. Find an example of a function for which

   (a) The left-hand, right-hand, and trapezoid estimates for $n = 4$ are all underestimates.

   (b) The left-hand, right-hand, and trapezoid estimates for $n = 4$ are all overestimates.

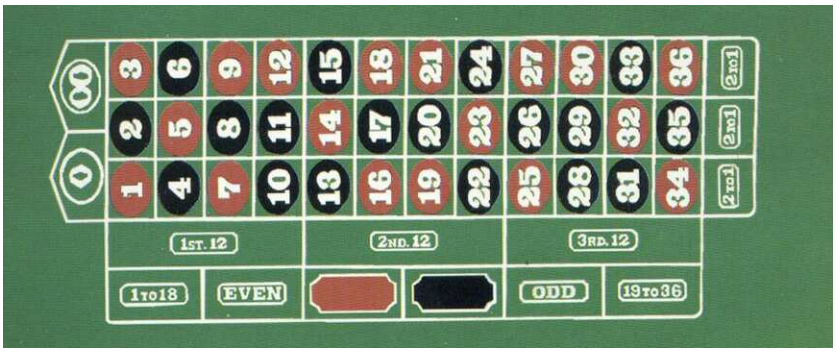
   (c) The LH and RH estimates are too high, but the trapezoid estimate is too low (again for $n = 4$).

5. Find the area of the finite region that is bounded by the y-axis, the line $y = 1$, and the graph of $y = x^{1/4}$ in two ways:

   (a) By integrating with respect to x and

   (b) By writing x as a function of y and integrating with respect to y.

6. Let's calculate some probabilities for Roulette.



   (a) Suppose I put a chip on "3". That means I win if and only if the ball lands on 3. What is the probability that I win?

   (b) Fill in the table to the right with probabilities of winning the bets shown.

   (c) Suppose I keep putting a dollar chip on "red" all night long, say for 200 games. On average, how many times will I win?

   (d) The red bet pays 1:1, meaning that if I win, then I get my original dollar back, plus one more dollar. If I start the night with \$200, what is the least I can have at the end? What's the most? What will I have on an average night?

Bet	**Prob**
1 or 2	
1 or 2 or 4 or 5	
odd	
red	
both odd and red	
either odd or red	