Douglass Houghton Workshop, Section 1, Tue 2/22/2012

Worksheet Jamboree

With a lot of hard work, we filled in the table to the right with the values of $\int_{-\pi}^{\pi} f(x)g(x)\,dx$, where f is the row and g is the column.

	1	$\sin(nx)$	$\cos(nx)$
1	2π	0	0
$\sin(mx)$	0	$\begin{cases} \pi & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}$	0
$\cos(mx)$	0	0	$\begin{cases} \pi & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}$

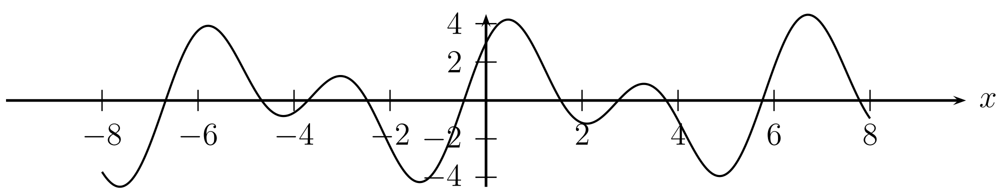
The implication was that for a function of the form

$$\begin{aligned} f(x) = a_0 &+ a_1\cos(x) + a_2\cos(2x) + a_3\cos(3x) + \cdots \\ &+ b_1\sin(x) + b_2\sin(2x) + b_3\sin(3x) + \cdots \end{aligned}$$

integrating against a sine or cosine function makes almost all the terms 0, so

$$\int_{-\pi}^{\pi} f(x)\,dx = 2\pi a_0, \quad \int_{-\pi}^{\pi} f(x)\cos(nx)\,dx = \pi a_n, \quad \text{and} \quad \int_{-\pi}^{\pi} f(x)\sin(nx)\,dx = \pi b_n.$$

1. Consider the function $h(x)$ whose graph looks like this:



Suppose you do some numerical integration and discover that

$$\begin{aligned} \int_{-\pi}^{\pi} h(x)\,dx &= 0 & \int_{-\pi}^{\pi} h(x)\cos(2x)\,dx &= 6.28 \\ \int_{-\pi}^{\pi} h(x)\cos(x)\,dx &= 3.14 & \int_{-\pi}^{\pi} h(x)\sin(2x)\,dx &= 4.71 \\ \int_{-\pi}^{\pi} h(x)\sin(x)\,dx &= 6.28 & \int_{-\pi}^{\pi} h(x)\cos(nx)\,dx &= \int_{-\pi}^{\pi} h(x)\sin(nx)\,dx = 0 \text{ for } n \geq 3. \end{aligned}$$

Can you guess a formula for $h(x)$? Use what you know!

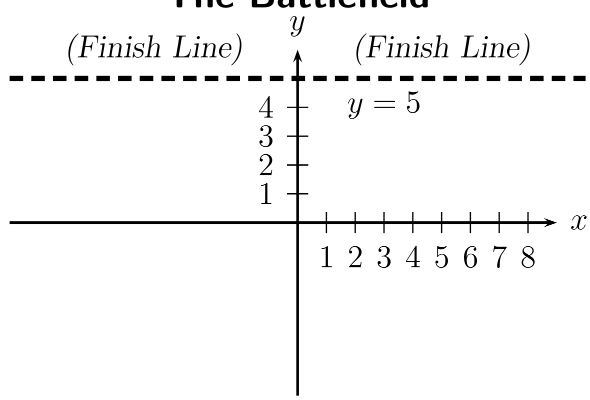
2. Help Barbie to become pink! If t is a number between 0 and 1 which represents the current grayness of a pixel ($0 =$ black, $1 =$ white), come up with three functions $r(t)$, $g(t)$, and $b(t)$ which give the red, green, and blue values for what the pixel *should* look like.

   Progress: : We established that the RGB values for red and white are $(1, 0, 0)$ and $(1, 1, 1)$ respectively, so it seems natural that pink should be the average of them, namely $(1, .5, .5)$. Since we want black to become pink and white to stay white, we have

t	$r(t)$	$g(t)$	$b(t)$
0.0	1.0	0.5	0.5
1.0	1.0	1.0	1.0

3. (This problem appeared on a Winter, 2003 Math 116 exam) The newest FOX reality show, "BattleBugs: Clash of the Beetles" begins (at $t = 0$) with eight assorted insects placed randomly on a large mat (the "battlefield", pictured here), on which is marked a "finish line". The producers hoped that the bugs would battle to be first to cross the finish line, but instead they wander around, each according to its nature. The motion of each bug is described by the equations below. Both x and y are measured in inches.

The Battlefield



Hercules Beetle $x(t) = \cos(t/2)$ $y(t) = \sin(t/2)$	**Ladybug** $x(t) = e^{-t}$ $y(t) = e^{-2t}$	**Tiger Beetle** $x(t) = 1 + t$ $y(t) = -1 + 8t$	**Longhorned Beetle** $x(t) = 3 + t$ $y(t) = 4 - t$
Dung Beetle $x(t) = t$ $y(t) = -2$	**Scarab** $x(t) = 2 - 7t$ $y(t) = -1 - 7t$	**June Beetle** $x(t) = 0$ $y(t) = -1$	**African Ground Beetle** $x(t) = \sin(t)$ $y(t) = \cos(t)$

Which bug (or bugs)...

(a) move repetitively?

(b) move fastest?

(c) will move very slowly (or not at all), in the long run?

(d) begin closest to the finish line?

(e) will reach the finish line first?

(f) gets the dizziest?