Douglass Houghton Workshop, Section 2, Thu 2/20/2012

Worksheet In it to Win it

With a lot of hard work, we have filled in almost all of the table to the right with the values of $\int_{-\pi}^{\pi} f(x)g(x)\,dx$, where f is the row and g is the column.

	1	$\sin(nx)$	$\cos(nx)$
1	2π	0	0
$\sin(mx)$	0	$\begin{cases} \pi & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}$	0
$\cos(mx)$	0	0	

1. The only thing missing is the lower right corner. Integrate both sides of

$$\cos((m+n)x) = \cos(mx)\cos(nx) - \sin(mx)\sin(nx)$$

and use what we already know to find $\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\,dx$. Remember that m and n are positive integers.

2. (After #1) Let $h(x) = 5 + \sin(x) + 2\cos(x) + 3\sin(2x) - 5\cos(2x)$.

   (a) Use your calculator to compute:

$$\int_{-\pi}^{\pi} h(x)\,dx =$$
$$\int_{-\pi}^{\pi} h(x)\sin(x)\,dx =$$
$$\int_{-\pi}^{\pi} h(x)\cos(x)\,dx =$$
$$\int_{-\pi}^{\pi} h(x)\sin(2x)\,dx =$$
$$\int_{-\pi}^{\pi} h(x)\cos(2x)\,dx =$$

   (b) Explain the results using the table above.

3. Last time we guessed correctly that

$$V_C = R_0\sqrt{\frac{g}{R_0+h}}.$$

is the velocity needed to achieve a circular orbit at height h above the surface of the earth, where

$$R_0 = \text{the radius of the earth } (6371\,\text{km}), \text{ and}$$
$$g = \text{the acceleration due to gravity } (9.8\,\text{m/s}^2).$$

   (a) So how long does it take to orbit the earth at height h?

   (b) How high would you have to be in order for it to take 24 hours to make one orbit?

   (c) What would it be like to orbit at that height around the equator?

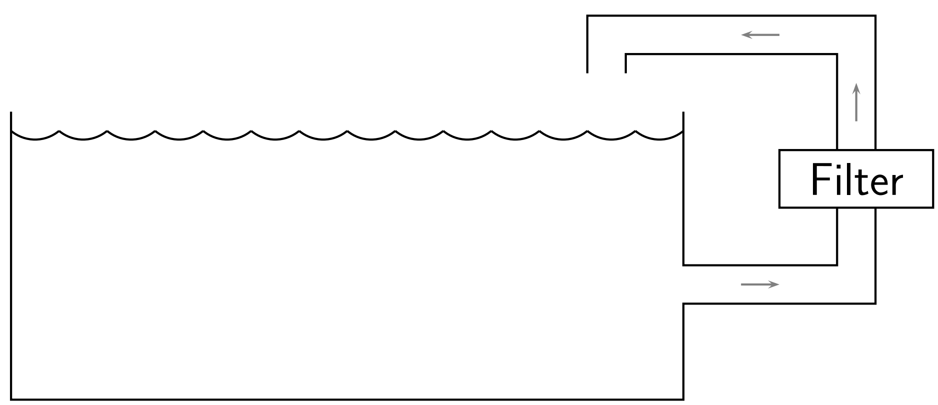
Remember what we found when computing the probabilitiy of the "Hard Eight" bet in craps:

$$\text{Prob of winning in } n \text{ rolls} = W + WC + WC^2 + \cdots + WC^{n-1} = W\frac{1-C^n}{1-C}$$

$$\text{Prob of winning} = W + WC + WC^2 + WC^3 + \cdots = \frac{W}{1-C}.$$

4. Find the probability of winning the "Pass" bet in craps.

5. (This problem appeared on a Fall, 2011 Math 116 Exam) An aquarium containing 100 liters of fresh water will be filled with a variety of small fish and aquatic plants. A water filter is installed on the tank to help remove the ammonia produced by the decomposing organic matter generated by plants and fish in the aquarium. The filter takes water from the tank at a rate of 20 liters every hour. The water is then filtered and returned to the aquarium at the same rate of 20 liters every hour. Ninety percent of the ammonia contained in the water that goes through the filter is removed. It is estimated that the fish and plants produce 30 mg of ammonia every hour. Assume the ammonia mixes instantly with the water in the aquarium.



   (a) Let $Q(t)$ be the amount in mg of ammonia in the fish tank t hours after the fish were introduced into the aquarium. Find the differential equation satisfied by $Q(t)$. Include its initial condition.

   (b) Find the amount of ammonia in the fish tank 3 hours after the fish were introduced into the aquarium. Include units.

   (c) What happens to the value of $Q(t)$ in the long run?

6. Help Barbie to become pink! If t is a number between 0 and 1 which represents the current grayness of a pixel ($0 =$ black, $1 =$ white), come up with three functions $r(t)$, $g(t)$, and $b(t)$ which give the red, green, and blue values for what the pixel *should* look like.