Douglass Houghton Workshop, Section 1, Wed 2/15/2012

Worksheet How far that little candle throws its beams!

1. Find the probability of winning the "Hard Eight" bet in craps.

   Progress: We have shown that the probability of winning on the ith roll is $\left(\frac{1}{36}\right)\left(\frac{25}{36}\right)^{i-1}$, and we know we need to add those up to get the total probability.

   (a) Let $W = \frac{1}{36}$ and $C = \frac{25}{36}$. So the probability of winning within the first n rolls is

   $$P_n = W + WC + WC^2 + WC^3 + \cdots + WC^{n-1}.$$

   Write down the sum for CP_n.

   (b) So what is $P_n - CP_n$?

   (c) Now find an expression for P_n that doesn't contain "$\sum$" or "$\cdots$".

   (d) Take the limit as $n \to \infty$ of P_n to get the exact probability of winning the Hard Eight bet in *any* number of rolls.

2. The Mercury memorial has this equation on it:

   $$V_C = R_0\sqrt{\frac{g}{R_0 + h}}.$$

   What does it mean?

3. We've done a few integrals with sines and cosines. For reasons so far unexplained, we'd like to fill in this table:

	1	$\sin(nx)$	$\cos(nx)$
1	2π	0	0
$\sin(mx)$	0	0 if $m \neq n$	0
$\cos(mx)$	0	0	

   with the values of $\int_{-\pi}^{\pi} f(x)g(x)\,dx$, where f is the row and g is the column. We need $\int_{-\pi}^{\pi} \sin(mx)\sin(nx)\,dx$ where $m = n$, and also $\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\,dx$.

4. Consider an ecosystem with two species, roadrunners and coyotes. Their populations are represented by x and y, and they are changing according to these differential equations:

   $$\frac{dx}{dt} = 0.2x - 0.005xy \qquad \text{and} \qquad \frac{dy}{dt} = -0.5y + 0.01xy.$$

   (a) Which variable is roadrunners, and which is coyotes? Think of some rationale to explain the differential equations.

   (b) Draw a slope field for the system, for $0 \leq x \leq 100$, $0 \leq y \leq 100$. Describe the solution curves.

5. Recall the "Field" bet in craps.

   (a) Suppose we bet a dollar on Field, for 36 rolls in a row. In an average run, how many bets will we lose? How many will we win even money? How many will we win double?

   (b) How much money does the casino make, on average, on 36 bets?

6. (This problem appeared on the Fall, 2007 Math 116 Final Exam. Really!) Newton's law of cooling (or warming) says that the rate of change of the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding medium. Suppose that a thermometer used by a veterinarian to find the temperature of a sick horse obeys Newton's law of cooling. Further suppose that before insertion the thermometer reads 82° F, after one minute it reads 92°, and after another minute it reads 97° F, and that a sudden convulsion unexpectedly destroys the thermometer after the 97° reading. Call the horse's temperature T_h.

   (a) Write a differential equation for the temperature T (a function of time t) of the thermometer. Your equation may involve the constant T_h.

   (b) Solve the differential equation for T to find a general solution for T. Your solution may include undetermined constants such as T_h.

   (c) Use the temperature data to solve for T.