Douglass Houghton Workshop, Section 1, Wed 1/25/2012

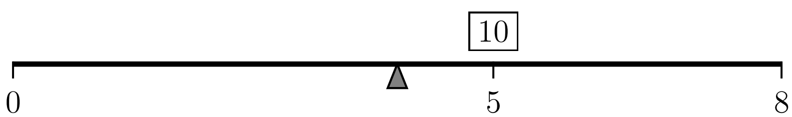
Worksheet Freddie

1. There is *still* nothing special at latitude $14°38'53''$ N, longitude $78°6'28''$ W. It's just a point in the ocean. But, if you were to shoot a neutrino from the middle of the Diag (latitude $42°16'36''$ N, longitude $83°44'15''$ W) to that point, through the earth's crust, its deepest point would be directly under a very interesting place. Find that place, and how deep the neutrino is there.

   Last time we worked on converting latitude ϕ, longitude θ, radius r to cartesian coordinates. We got as far as $z = r\sin\phi$. We tried to make a similar argument for x and y, but it didn;t seem to work out.

   Define the x- and y-axes, and find a way to compute x and y from ϕ and θ. Good idea from last time: Look down on the earth from above the North Pole.

2. Imagine an 8-foot long board sitting on a pivot, with a 10 lb bag of flour one foot to the right of center:



   The flour will make the board want to rotate clockwise. We say it creates a **torque** on the board. Increasing the weight makes the torque stronger, as does moving the flour to the right. So we say the strength of the torque is

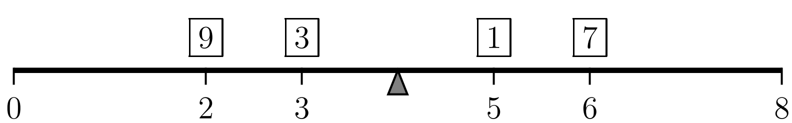
$$T = r \times F$$

   where r is the distance from the pivot to the flour, and F is the weight of the flour. So in this case the torque is

$$T = r \times F = (1\text{ft}) \times (10\text{lb}) = 10\text{ft} \cdot \text{lb}.$$

   (a) A weight on the left side of the board will generate a negative (counterclockwise) torque. Where should you place a 3 lb bag of ice to balance out the flour? That's the same as making the total torque equal to 0.

   (b) Whats the total torque of the weights shown here?

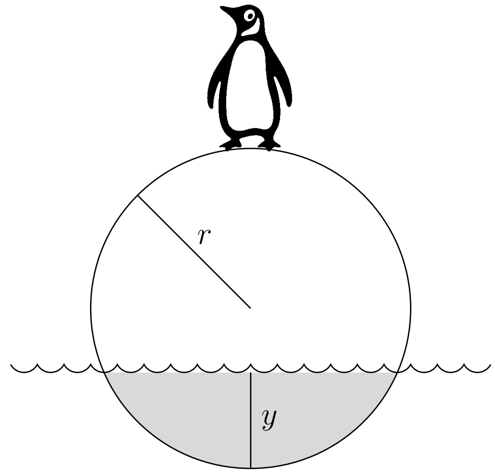


   (c) Where should we place the pivot to make the board in part (b) balance?

3. Evaluate $\int_{-\pi}^{\pi} \sin(mx)\sin(nx)$ where m and n are positive integers. You may want to use a trig identity or two from the front of your book.

4. The buoyancy force on a floating object is proportional to the volume of water it displaces. Using this fact, Sierra plans to find the weight of a penguin, whose name is "Freddie", by floating beachballs on the ocean and enticing Freddie to climb on top. She will measure the depth the ball sinks to, and thereby deduce the penguin's weight.

   So given a beachball of radius r which is partially submerged in the water, find a formula for the volume of the ball which is below the water line when its bottom is at depth y. Check that your formula makes sense for the values $y = 0$, $y = r$, and $y = 2r$.



5. Suppose you are pumping water up from a lake to a water tank. The tank is a rectangular solid, with a base that is $46'' \times 38\frac{1}{2}''$, and a height of $38''$. The base is 27 feet above the lake. Water weighs $62.4\,\mathrm{lb/ft}^3$.

   (a) How much work, in ft·lb, will it be to fill the tank?

   (b) It took about 10 oz of gasoline to pump the water up. A gallon of gasoline contains about 132 megajoules of energy, according to Wikipedia. Use the fact that 1 gallon is 128 ounces and 1 ft·lb is 1.355 joules to find the efficiency of the pump.

6. Below are the graphs of several functions $f(x)$, $g(x)$, $h(x)$, $i(x)$, $j(x)$, and $k(x)$. Do not assume that the y-axis scales on these graphs are equal or even comparable. We have calculated LEFT(6), RIGHT(6), TRAP(6), and MID(6) for four of these six functions. Label each column with the name of the function estimated in that column.

Function:				
LEFT(6):	64.2	.328	.255	80.0
RIGHT(6):	65.8	.444	.421	80.0
TRAP(6):	65.0	.386	.338	80.0
MID(6):	65.0	.388	.331	80.0