Douglass Houghton Workshop, Section 1, Mon 1/23/2012

Worksheet Escargo

1. There is still nothing special at latitude 14°38′53″ N, longitude 78°6′28″ W. It's just a point in the ocean. But, if you were to shoot a neutrino from the middle of the Diag (latitude 42°16′36″ N, longitude 83°44′15″ W) to that point, through the earth's crust, its deepest point would be directly under a very interesting place. Find that place, and how deep the neutrino is there.

   Progress:

   - Averaging the latitudes and longitudes doesn't work, but
   - If we could find the cartesian coordinates for the two points, then we could average them and get the answer.

2. Let's practice some integration by parts.

   (a) $\int xe^x\,dx$

   (b) $\int \ln x\,dx$

   (c) $\int e^x \sin x\,dx$

   (d) $\int_0^1 \tan^{-1}(x)\,dx$ Hint: $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$

3. Last time we worked on finding

$$\int_{-\pi}^{\pi} \sin(mt)\cos(mt)\,dt$$

   where m and n are positive integers. One idea is to look at some examples (e.g., $m = 1, n = 1$, $m = 2, n = 3$, etc.) Another is to use trig identities to make that product into something we can integrate. There are two identities in the front of your book which might help:

$$\sin(x+y) = \sin x\cos y + \cos x\sin y$$
$$\sin(x-y) = \sin x\cos y - \cos x\sin y$$

   See if you can use them to get the integral.

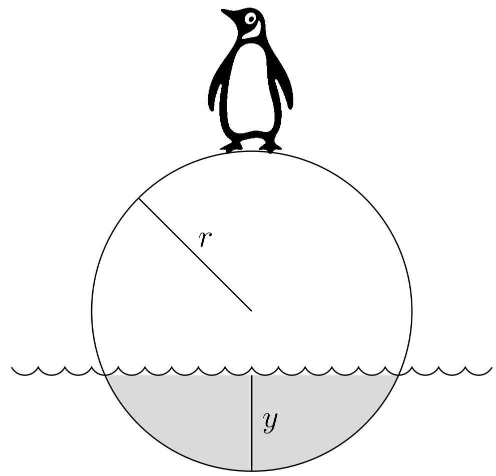
4. Think of a similar trick to approach

$$\int_{-\pi}^{\pi} \sin(mt)\sin(nt)\,dt$$

   where m and n are still positive integers.

5. The buoyancy force on a floating object is proportional to the volume of water it displaces. Using this fact, a group of scientists plans to study the weights of penguins by floating beachballs on the ocean and enticing penguins to climb on top. They then measure the depth the ball sinks to, and thereby deduce the penguin's weight.

   So given a beachball of radius r which is partially submerged in the water, find a formula for the volume of the ball which is below the water line when its bottom is at depth y. Check that your formula makes sense for the values $y = 0$, $y = r$, and $y = 2r$.



6. Consider the **gamma function**: $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1}\, dt$, for $x > 0$.

   (a) Use integration by parts to prove that $\Gamma(x + 1) = x\Gamma(x)$.

   (b) Show that $\Gamma(1) = 1$. Then fill in this chart, using part (a):

x	1	2	3	4	5	6
$\Gamma(x)$						

   (c) So if x is a positive integer, what is $\Gamma(x)$?

7. Last time we found that if a parabola goes through $(-1, R)$, $(0, S)$, and $(1, T)$, then the area under it, from -1 to 1, is $\frac{1}{3}(R + 4S + T)$.

   (a) Suppose a parabola goes through $(0, R)$, $(1, S)$, and $(2, T)$. Find the area under it from 0 to 2.

   (b) Suppose a parabola goes through $(0, R)$, (h, S), and $(2h, T)$. What's the area under it from 0 to $2h$?

   (c) The left- and right-hand sums are ways to estimate the area under a curve, by assuming the curve is a step function. The trapezoid rule improves this by assuming the curve is piecewise linear. Derive an even better rule, by assuming the curve is a bunch of parabolas stuck together, and the function has values

x	a	$a+h$	$a+2h$	$a+3h$	$a+4h$	$\cdots$	$a+2nh$
$f(x)$	y_0	y_1	y_2	y_3	y_4	$\cdots$	y_{2n}.