Douglass Houghton Workshop, Section 1, Wed 1/18/2012

Worksheet Durmstrang

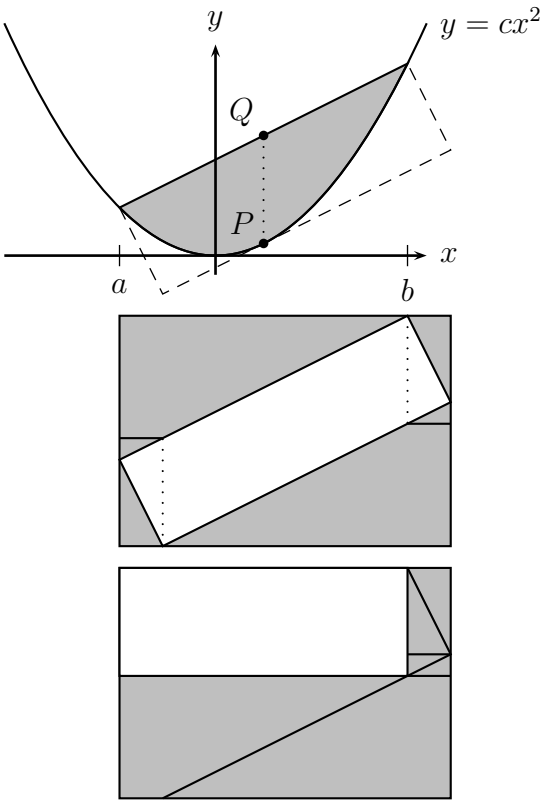
1. We need to resolve the parabolic lune problem. We know the lune has area

$$\frac{1}{6}c(b-a)^3.$$

and we wound that the point of tangency is

$$P = \left(\frac{b+a}{2}, c\left(\frac{b+a}{2}\right)^2\right).$$

What's the area of the box containing it? Consider the pictures on the right. How do the open areas compare to the area of the box?



2. Let's practice some substitution.

(a) $\int z(z+3)^{1/3}\,dz$

(b) $\int \dfrac{dx}{2+2\sqrt{x}}$

(c) $\int (z+2)\sqrt{1-z}\,dz$

(d) $\int \dfrac{3x-2}{\sqrt{2x+1}}\,dx$

3. Suppose that 1% of a certain population has hepatitis, and that a particular test for hepatitis is 98% accurate.

(a) Fill in the following table of possibilities. For instance, the upper-left corner is the probability that a randomly-chosen person has hepatitis *and* tests positive. The upper-right corner is the probability that a person has hepatitis *and* tests negative.

		Test Results	
		Positive	Negative
Has disease?	Yes		
	No		

(b) Given that a person tests positive, what is the probability that he or she actually has hepatitis?

4. There is nothing special at latitude $14°38'53''$ N, longitude $78°6'28''$ W. It's just a point in the ocean. But, if you were to shoot a neutrino from the middle of the Diag (latitude $42°16'36''$ N, longitude $83°44'15''$ W) to that point, through the earth's crust, its deepest point would be directly under a very interesting place. Find that place, and how deep the neutrino is there.

5. Suppose you have a function $f(x)$. You know:

   - f is a quadratic. That is, $f(x) = ax^2 + bx + c$ for some constants a, b, and c.
   - How to measure $f(-1)$, $f(0)$, and $f(1)$.

   You want to know $\int_{-1}^{1} f(x)\,dx$.

   (a) Let R, S, and T be the values you measure for $f(-1)$, $f(0)$, and $f(1)$, respectively. Find a formula for $f(x)$. (That is, find a, b, and c in terms of R, S, and T.)

   (b) Find $\int_{-1}^{1} f(x)\,dx$ in terms of the R, S, and T.

6. (Beads)

   (a) Suppose that Danielle has 5 black beads, all identical, and 4 white beads, also identical. How many different strings of beads can she make?

   (b) Suppose that Danielle has b black beads and w white beads, and $S(b, w)$ is the number of strings she can make. Find:

      i. $S(0, w)$ and $S(1, w)$ if $w > 0$

      ii. $S(b, 0)$ and $S(b, 1)$ if $b > 0$

   (c) Suppose w and b are both at least 1. How many strings are there whose last bead is white? How many whose last bead is black? (Answer in terms of the S notation from part (b).)

   (d) Suggest a way of computing $S(b, w)$.

7. Evaluate $\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\,dx$ where m and n are positive integers. (You might want to graph a few examples.)