Douglass Houghton Workshop, Section 1, Wed 1/11/12

Worksheet Casino

1. Find a nice formula for the volume of a cup with radii r and R, and height H. We were very close last time.

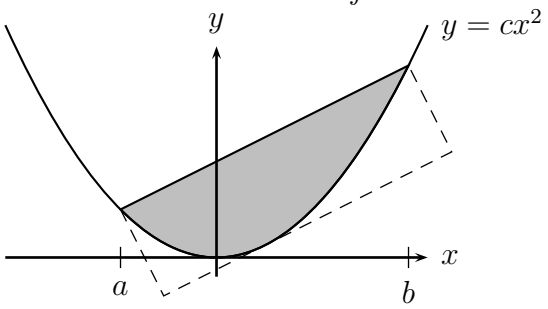
2. Find the exact area under one arch of the graph of $f(x) = A\sin(Bt)$, where $A, B > 0$.

3. Use one of the trig identities in the front of your textbook to compute $\int \sin^2(x)\,dx$.

4. We found the area of the shaded region of the parabola to be

   $$\frac{1}{6}c(b-a)^3.$$

   Now find the area of the rectangle containing it.



5. The **expectation** of a particular bet on a particular game is the average amount you'll win or lose. Calculate the expectation of

   (a) Betting \$1 on red in Roulette.

   (b) The Michigan Lottery's pick-3 game. It costs a dollar to buy a ticket, on which you pick a three digit number. If you pick the right one they pay you \$500 for your ticket.

   (c) Kazim's Roulette strategy: Put n \$1 chips on n different numbers.

   (d) Jeff's Roulette strategy: Bet \$1 on red, and quit if you win. Otherwise bet \$2, and quit if you win. Otherwise bet \$4, and quit if you win. Assume you start with \$7 in your pocket, and must go home if you're broke.

6. Find $\dfrac{d}{dx}\displaystyle\int_{\cos x}^{3} e^{t^2}\,dt$.

7. For $x > 1$, let $F(x) = \displaystyle\int_{x}^{x^3} \frac{1}{t^2+1}\,dt$.

   (a) Draw a picture (or pictures) on the board that explains what the quantity $F(x)$ represents and how it changes as x changes.

   (b) Find $F'(x)$.

   (c) Find $\lim_{x\to\infty} F'(x)$, and explain what this means in terms of your picture in (a).

   (d) Write an expression for $F(x)$ that does not involve an integral sign.

   (e) Find $\lim_{x\to\infty} F(x)$, and explain what this means in terms of your picture in (a).

8. Suppose you have a function $f(x)$. You know:

   - f is a quadratic. That is, $f(x) = ax^2 + bx + c$ for some constants a, b, and c.
   - How to measure $f(0)$, $f(1)$, and $f(2)$.

   You want to know $\int_0^2 f(x)\,dx$.

   (a) Let y_0, y_1, and y_2 be the values you measure for $f(0)$, $f(1)$, and $f(2)$. Find a formula for $f(x)$. (That is, find a, b, and c in terms of y_0, y_1, and y_2.)

   (b) Find $\int_0^2 f(x)\,dx$ in terms of the y's.

9. Here's another offering from the Michigan Lottery.

   Classic Lotto 47 In this game there is one bin containing 47 balls, numbered from 1 to 47. 6 balls are chosen from the bin. Once a ball has been drawn, it is not put back into the bin. For instance, last Saturday the winning numbers were `18-21-28-29-31-39`.

   (a) How many different possibilities are there for the first ball drawn?

   (b) Once the first ball has been drawn, how many possibilities are there for the second ball?

   (c) How many possibilities are there for the first two balls, if the order in which they are drawn matters? The first 3 balls? How about for all 6 balls?

   One catch, though: the order DOESN'T matter. In other words, the two drawings `18-21-28-29-31-39` and `21-31-28-29-39-18` are exactly the same as far as the game goes.

   (d) How does that affect the number of possible outcomes? Repeat part (c), taking into account that the order in which balls are drawn is not important.