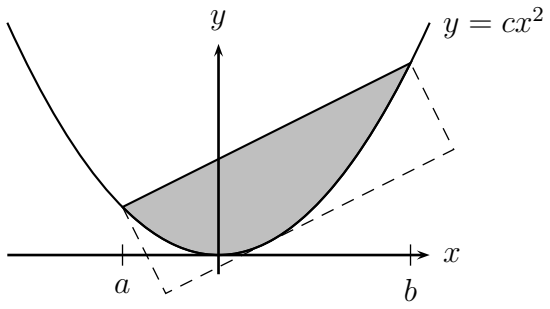
Douglass Houghton Workshop, Section 1, Mon 1/9/12

Worksheet Batrachomyomachia

1. Find the volume of a plastic cup, using any method you like. What assumptions do you need to make? What measurements? Avoid numbers.

2. (a) Find the area of the shaded region in the picture to the right. Make the answer as simple as possible.



Progress:

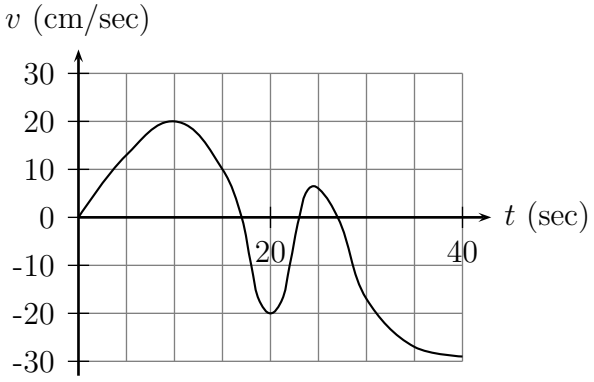
- The endpoints of the secant line are (a, ca^2) and (b, cb^2).
- So the slope of the secant line is $\frac{cb^2-ca^2}{b-a}$.
- So the area is $\int_a^b (\text{eqn of line} - cx^2)\,dx$.

   (b) Find the area of the dashed rectangle, which is tangent to the curve.

3. Find the interval on which the graph of $f(x) = \displaystyle\int_0^x \frac{1}{1+t+t^2}\,dt$ is concave up.

4. (This is derived from Problem 47 on page 295 of your book.)

A mouse is trapped in a psychologist's experiment. She moves back and forth in a straight tunnel. The cruel experimenter attracts the mouse with bits of cheese at one end or the other. Sometimes he also puts a frog in the tunnel to scare the mouse away, because mice are terrified of frogs. The graph of the mouse's velocity, v, is given to the right, with a positive velocity corresponding to motion toward the right end.



Tell the story from the mouse's point of view. You might write it as a timeline, explaining what happened when. Make up explanations for all the significant features of the graph.

5. The breathing of a frog is cyclic, and when it is relaxed (because no mice are around), the time from beginning of inhalation to end of exhalation is about 5 seconds. The maximum rate of air flow into the lungs is about 50 milliliters per second.

   (a) Write a trigonometric function that models the rate of air flow into the lungs.

   (b) Use this function to find the maximum amount of inhaled air in the lungs.

6. Kazim suggests that a strategy for playing Roulette is to put a \$1 chip on half of the numbers on the board (19 chips).

   (a) What is the probability that one of his bets will win? If it does, he wins \$35.

   (b) If he plays 200 games this way, what is the most he might lose, and the most he might win?

   (c) What will happen on an "average" night?

7. Here's another idea for making money at Roulette, proposed by a student in a previous DHSP class. We'll call him "Jeff".

   (1) Bet \$1 on red. If it wins, quit, having won 1 dollar. Otherwise...

   (2) Bet \$2 on red. If it wins, quit, having won $2 - 1 = 1$ dollar. Otherwise...

   (3) Bet \$4 on red. If it wins, quit, having won $4 - 2 - 1 = 1$ dollar. Otherwise...

   (4) Bet \$8 on red. If it wins, quit, having won $8 - 4 - 2 - 1 = 1$ dollar. Otherwise...

   And so on. Jeff argues that in each case he will win a dollar, so this is a foolproof strategy for making money. Recall that the probability of winning the red bet is $p = 9/19$, and the probability of losing is $q = 10/19$.

   Suppose Jeff starts with 7 dollars in his pocket. Will he always make money?

8. The Michigan Lottery offers several exciting and fun ways to spend money. Let's calculate the odds of one of them.

   Daily 3 Three bins, numbered 1, 2, and 3, each contain ten ping-pong balls, numbered 0 through 9. A ball is chosen from each bin, so that the result of the drawing is a 3-digit number. Players likewise choose a 3-digit number to play.

   (a) What is the probability of getting all three digits correct?

   (b) You can also play your numbers "boxed". That means that if you match the three digits *in any order*, you win. What is the probability of winning a boxed ticket? Does it depend on what numbers you play?