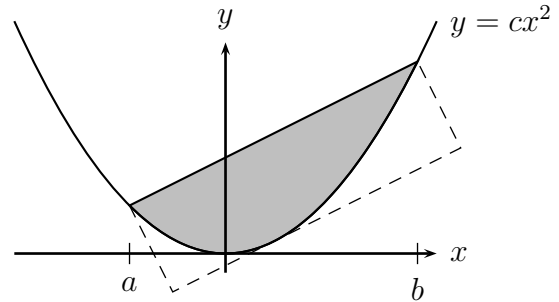


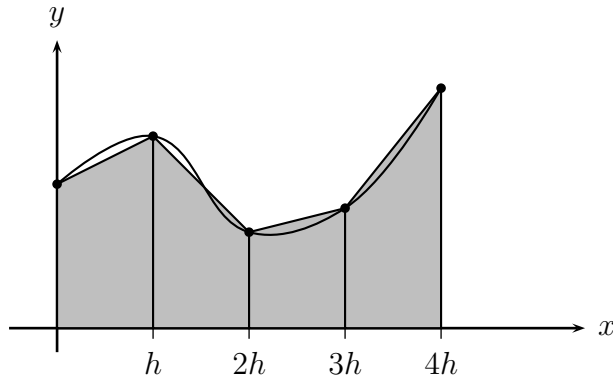
## Worksheet Archimedes

- Find the volume of a plastic cup, using any method you like. What assumptions do you need to make? What measurements?

- Find the area of the shaded region in the picture to the right. Make the answer as simple as possible.
  - Find the area of the dashed rectangle, which is tangent to the curve.



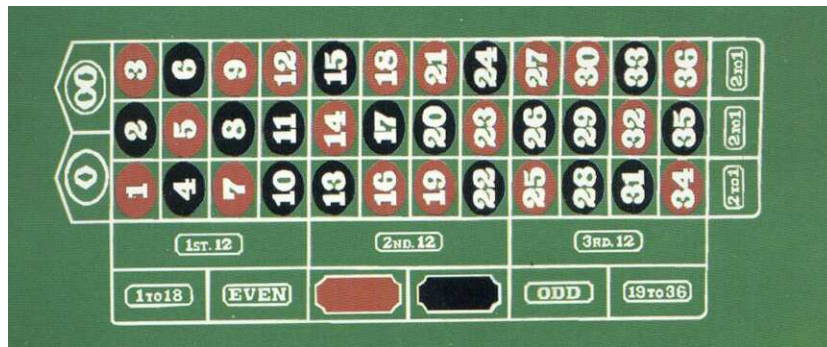
- Last fall we talked a lot about left-hand and right-hand sums for approximating integrals. You may have thought to yourself: “Rectangles seem like a very poor approximation for the area under a curve. There’s all that wasted space at the top. Can’t we do better?” The truth is that rectangles are the simplest way to approximate a definite integral, but we *can* do better. The simplest improvement would be to use trapezoids instead of rectangles:



Suppose we have  $f(0) = y_0, f(h) = y_1, \dots, f(4h) = y_4$ .

- Write a formula in terms of the  $y$ 's for the left-hand sum approximation of  $\int_0^{4h} f(x)dx$  with 4 rectangles.
- Write a similar formula for the right-hand sum approximation of  $\int_0^{4h} f(x)dx$ .
- Find the area of the leftmost trapezoid in terms of  $h, y_0$ , and  $y_1$ .
- Write a formula for the trapezoid approximation of  $\int_0^{4h} f(x)dx$ , with 4 trapezoids. How is it related to the formulas you found in (a) and (b)?
- How can you do better than trapezoids? Hint: First we approximated the curve with constant functions, then with line segments. What could we try next?

4. Find an example of a function for which
- The left-hand, right-hand, and trapezoid estimates for  $n = 4$  are all underestimates.
  - The left-hand, right-hand, and trapezoid estimates for  $n = 4$  are all overestimates.
  - The LH and RH estimates are too high, but the trapezoid estimate is too low (again for  $n = 4$ ).
5. Find the area of the finite region that is bounded by the  $y$ -axis, the line  $y = 1$ , and the graph of  $y = x^{1/4}$  in two ways:
- By integrating with respect to  $x$  and
  - By writing  $x$  as a function of  $y$  and integrating with respect to  $y$ .
6. Let's calculate some probabilities for Roulette.



- Suppose I put a chip on “3”. That means I win if and only if the ball lands on 3. What is the probability that I win?
- Fill in the table to the right with probabilities of winning the bets shown.
- Suppose I keep putting a dollar chip on “red” all night long, say for 200 games. On average, how many times will I win?
- The red bet pays 1:1, meaning that if I win, then I get my original dollar back, plus one more dollar. If I start the night with \$200, what is the least I can have at the end? What's the most? What will I have on an average night?

Bet	Prob
1 or 2	
1 or 2 or 4 or 5	
odd	
red	
both odd and red	
either odd or red	