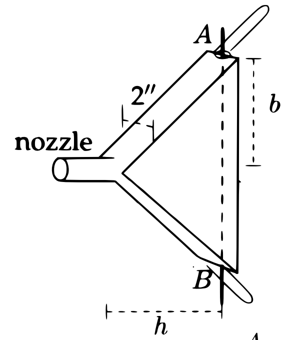
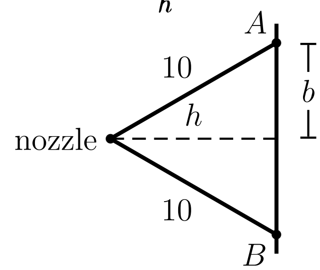
Douglass Houghton Workshop, Section 2, Tue 12/05/23

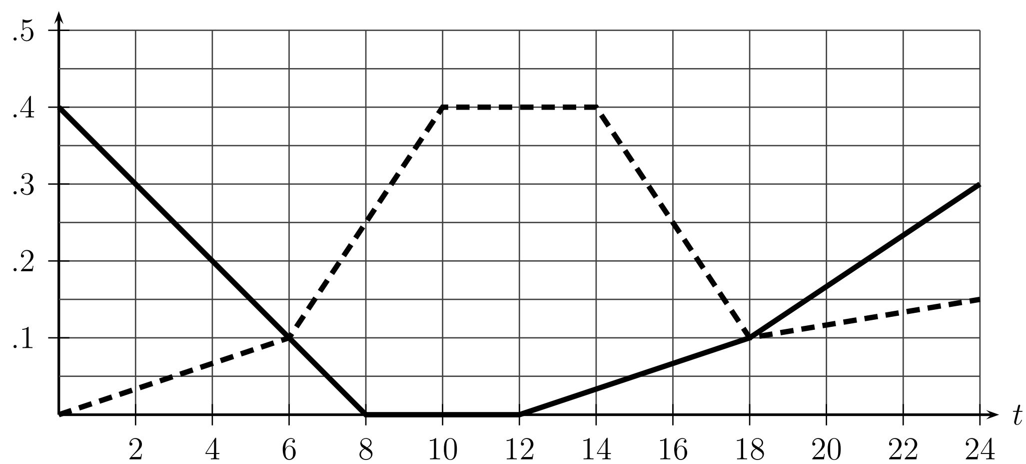
Worksheet To Infinity, and Beyond

1. (This problem appeared on a Winter, 2008 Math 115 exam) A bellows has a triangluar frame made of three rigid pieces. Two pieces, each 10 inches long, are hinged at the nozzle. They are attached to the third piece at points A and B which can slide, as shown in the diagrams below. (The figures show a 3D sketch of the bellows and a 2D sketch that may be specifically useful to solve the problem.)
   Each piece of the frame is 2 inches wide, so the volume (in cubic inches) of air inside the bellows is equal to the area (in square inches) of the triangluar cross-section above, times 2. Suppose you pump the bellows by moving A downward toward the center at a constant speed of 3 in/sec. (So B moves upwards at the same speed.) What is the rate at which air is being pumped out when A and B are 12 inches apart? (So A is 6 inches from the center of the vertical piece of the frame.)





2. (Fall, 2011) Suppose the graph below shows the rate of snow melt and snowfall on Mount Arvon, the highest peak in Michigan (at a towering 1970 ft), during a day (24 hour period) in April of last year. The function $m(t)$ (the solid curve) is the rate of snow melt, in inches per hour, t hours after the beginning of the day. The function $p(t)$ (the dashed curve) is the snowfall rate in inches per hour t hours after beginning of the day. There were 18 inches of snow on the ground at the beginning of the day.



   (a) Over what period(s) was the snowfall rate greater than the snow melt rate?

   (b) When was the amount of snow on Mount Arvon increasing the fastest? When was it decreasing the fastest?

   (c) When was the amount of snow on Mount Arvon the greatest? Explain.

   (d) How much snow was there on Mount Arvon at the end of the day (at $t = 24$)?

   (e) Sketch a well-labeled graph of $P(t)$, an antiderivative of $p(t)$ satisfying $P(0) = 0$. Label and give the coordinates of the points on the graph of $P(t)$ at $t = 10$ and $t = 18$.

3. (From the Winter, 2007 Math 115 Final Exam) Suppose that f and g are continuous functions with

$$\int_0^2 f(x)\,dx = 5 \qquad \text{and} \qquad \int_0^2 g(x)\,dx = 13.$$

Compute the following. If the computation cannot be made because something is missing, explain clearly what is missing.

(a) $\displaystyle\int_4^6 f(x-4)\,dx$

(b) $\displaystyle\int_{-2}^0 2g(-t)\,dt$

(c) $\displaystyle\int_2^0 (f(y)+2)\,dy$

(d) $\displaystyle\int_2^2 g(x)\,dx$

(e) Suppose that f is an even function. Find the average value of f from -2 to 2.

4. In Summer 2018, it was reported that ocean levels were rising at the rate of $\frac{1}{2}$ mm per year, and that that rate had tripled over the previous decade. If that tripling continues, how long will it be until the sea level has risen half a meter? 1 meter?

5. (Fall, 2013) Alyssa is running sprints in Crisler Center. She begins in the middle of the "M" at the center of the court and runs north and south. Her velocity, in meters per second, for the first 9 seconds is $v(t) = t\sin(\frac{\pi}{3}t)$, where t is the number of seconds since she started running. She is running north when $v(t)$ is positive and south when $v(t)$ is negative.

(a) Show that $f(t) = \dfrac{9}{\pi^2}\sin\left(\dfrac{\pi}{3}t\right) - \dfrac{3}{\pi}t\cos\left(\dfrac{\pi}{3}t\right)$ is an antiderivative of $v(t)$.

(b) Where on the court is Alyssa after 9 seconds?

(c) What is the total distance traveled by Alyssa in 9 seconds?

6. Sofia is studying a colony of *campylobacter jejuni* bacteria. She finds that the growth rate of the colony is increasing exponentially. That is, if $P(t)$ is the population in thousands after t hours, then $P'(t) = Ae^{kt}$ for some constants A and k.

(a) Suppose there are 1000 bacteria at the start of the experiment. Write an integral which gives the number of bacteria present after T hours.

(b) Use the Fundamental Theorem of Calculus to get a formula without an integral for the number of bacteria after T hours.

(c) Suppose the bacteria grew at an initial rate of 500 bacteria per hour, and after 6 hours the rate has increased to 1000 bacteria per hour. Find values for the constants A and k.

(d) How many bacteria are there 6 hours after the experiment started?

7. Briza also does an experiment with the same starting population of bacteria, but she plays Taylor Swift to the bacteria. She finds the bacteria LOVE Taylor Swift, and they grow while a song is playing and stop growing between songs. Briza plays them a series of 3-minute songs with 3-minute breaks between them, and finds that t hours after the experiment starts, the growth rate (in thousands per hour) is $1+\sin(20\pi t)$. How many bacteria grow in each 6-minute cycle?