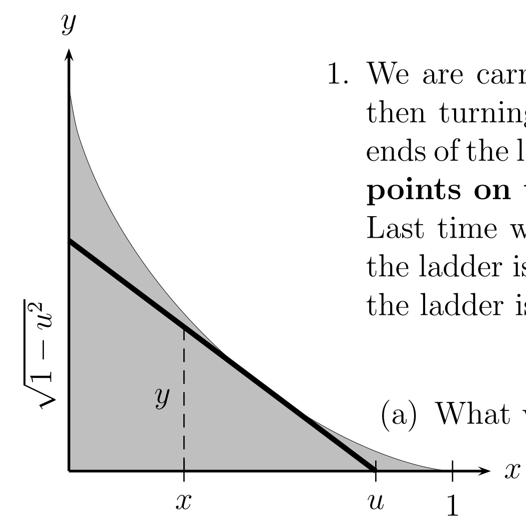
Douglass Houghton Workshop, Section 2, Thu 11/30/23

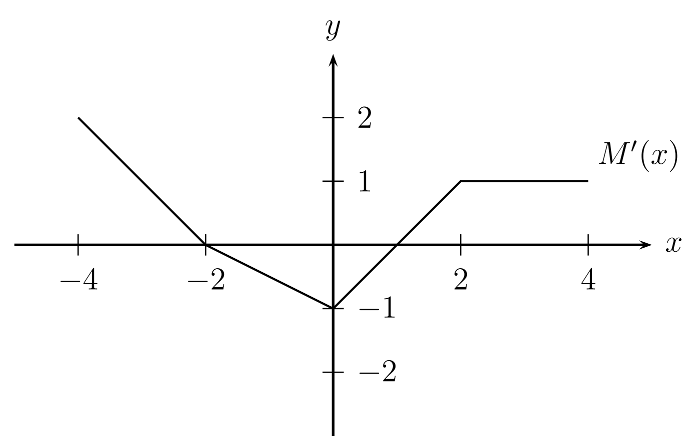
Worksheet Some Rise By Sin, and Some By Virtue Fall



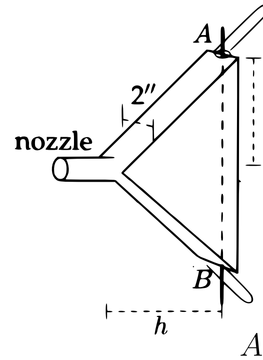
1. We are carrying a ladder of length 1 down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?**
   Last time we found that if $0 \le x \le u \le 1$ and the base of the ladder is at $(u, 0)$, then the distance from $(x, 0)$ north to the ladder is
   $$y = \frac{u-x}{u}\sqrt{1-u^2}.$$
   (a) What value of u maximizes y? (Keep x fixed!)
   (b) So for a fixed x, what is the maximum value of y, as the ladder moves?
   (c) You have found a formula for the curve at the top of the region we want. Simplify until it's beautiful. (This is the best part, so don't stop until it's truly wondrous.)

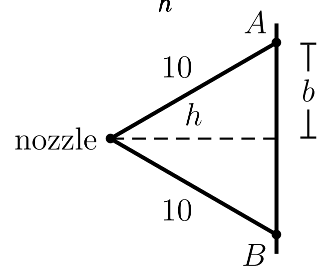
2. (Fall, 2011) For positive A and B, the force between two atoms is a function of the distance, r, between them:
   $$f(r) = -\frac{A}{r^2} + \frac{B}{r^3}.$$
   (a) Find the zeroes of f in terms of A and B.
   (b) Find all critical points and inflection points of f in terms of A and B.
   (c) If f has a local minimum at $(1, -2)$ find the values of A and B. Using your values for A and B, justify that $(1, -2)$ is a local minimum.

3. Here is the graph of the *derivative* of the continuous function $M(x)$. Using the fact that $M(-4) = -2$, sketch the graph of $M(x)$. Give the coordinates of all critical points, inflection points, and endpoints.



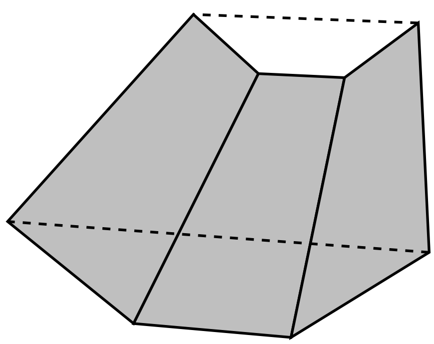
4. (This problem appeared on a Winter, 2008 Math 115 exam) A bellows has a triangluar frame made of three rigid pieces. Two pieces, each 10 inches long, are hinged at the nozzle. They are attached to the third piece at points A and B which can slide, as shown in the diagrams below. (The figures show a 3D sketch of the bellows and a 2D sketch that may be specifically useful to solve the problem.)

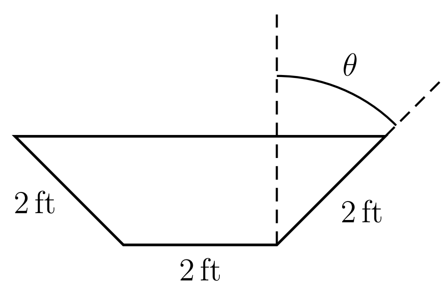




Each piece of the frame is 2 inches wide, so the volume (in cubic inches) of air inside the bellows is equal to the area (in square inches) of the triangluar cross-section above, times 2. Suppose you pump the bellows by moving A downward toward the center at a constant speed of 3 in/sec. (So B moves upwards at the same speed.) What is the rate at which air is being pumped out when A and B are 12 inches apart? (So A is 6 inches from the center of the vertical piece of the frame.)

5. A trough, as shown below, is to be made with a base that is 2 feet wide and 10 feet long. The sides of the trough are also 2 feet wide by 10 feet long, and are to be placed so they make an angle θ with the vertical.





(a) What is the area, in terms of θ, of a cross section of the trough perpendicular to its long side? What is the volume of the trough?

(b) What angle θ will give the trough the largest volume, and what is that volume? [Hint: you can always replace $\cos^2(\theta)$ with $1 - \sin^2(\theta)$.]