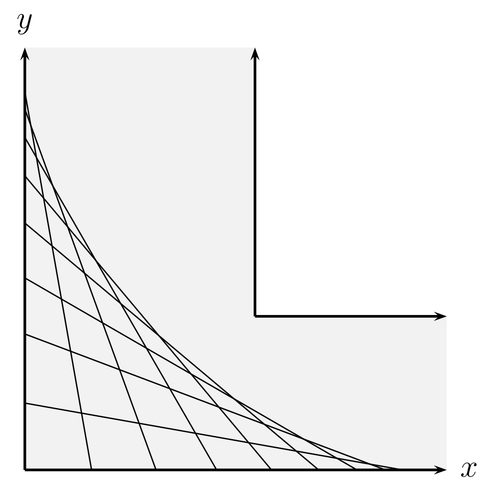
Douglass Houghton Workshop, Section 2, Tue 11/28/23

Worksheet Romeo, Romeo, Wherefore Art Thou Romeo?

1. Suppose we are carrying a ladder down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?** Let's assume the ladder has length 1. In the picture, the gray area is the hallway and the fine lines are the ladder in different positions.



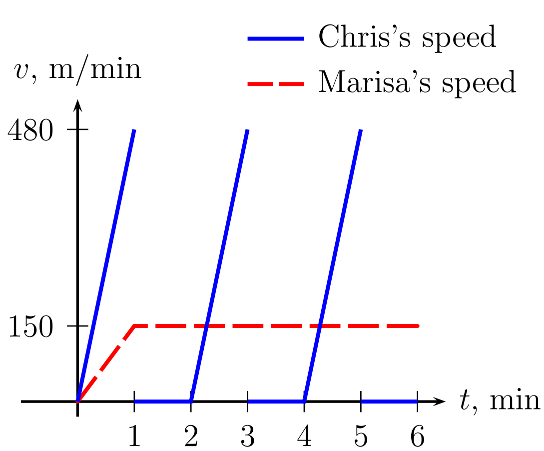
   (a) Suppose the base of the ladder is at the point $(u, 0)$. Where on the y-axis is the top of the ladder? Draw a picture!

   (b) Suppose you are standing at $(x, 0)$ and looking north (up the page). If $x < u$, how far away do you see the ladder?

   To be continued. . .

2. (Adapted from a Winter 2009 Math 115 Exam.) Marisa and Chris, after much friendly trash talk about who's the fastest runner, decide to have a race. The two employ very different approaches.

   - Marisa takes the first minute to accelerate to a slow and steady pace which she maintains through the remainder of the race.
   - Chris, on the other hand, spends the first minute accelerating to faster and faster speeds until he's exhausted and has to stop and rest for a minute—and then he repeats this process until the race is over. The graph below shows their speeds (in meters per minute), t minutes into the race. (Assume that the pattern shown continues for the duration of the race.)



   (a) What is Marisa's average speed over the first two minutes of the race? What is Chris's?

   (b) Chris immediately gets ahead of Marisa at the start of the race. How many minutes into the race does Marisa catch up to Chris for the first time?

   (c) Draw graphs of Chris's and Marisa's positions at time t. Be as precise as possible.

   (d) If the race is 720 meters total, who wins? What if it's 721 meters?

(Fall, 2014) Maya, as we know, is a popcorn afficianado. So after graduation she teams up with Ashley, who worked in a movie theater, to start a business growing and selling high-end popcorn to elite cinemas.

3. After careful study, Maya and Ashley have determined that they can produce up to 160 barrels of popcorn in a year. They can sell the first 100 barrels to movie theaters and any remaining popcorn to wholesalers. The revenue in dollars from from selling x barrels of popcorn will be

$$R(x) = \begin{cases} 6x & \text{if } 0 \leq x \leq 100 \\ 4x + 200 & \text{if } 100 < x \leq 160. \end{cases}$$

   (a) What is the price movie theaters pay for each barrel of popcorn?

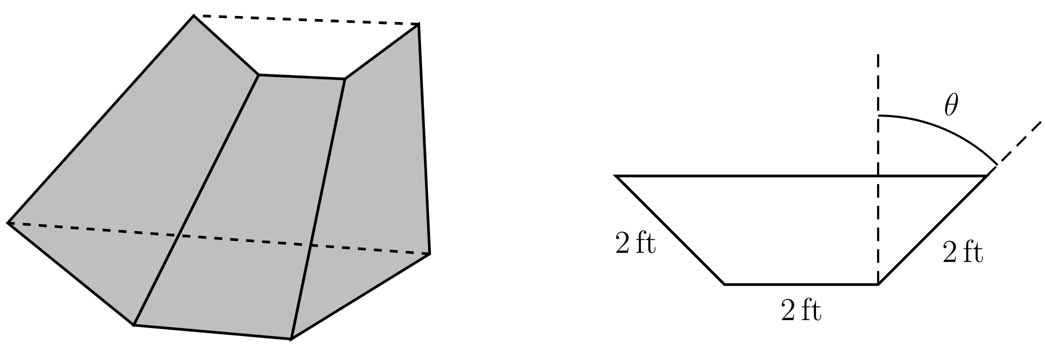
   (b) What is the price wholesalers pay?

   (c) It costs $C(x) = 20 + 3x + 24\sqrt{x}$ to produce x barrels of popcorn. (Use that formula for the rest of the problem.) What is the fixed cost of Maya and Ashley's operation?

   (d) At what production levels does marginal revenue equal marginal cost?

   (e) How many barrels of popcorn should Maya and Ashley produce to maximize their profit, and what is the maximum possible profit?

4. A trough, as shown below, is to be made with a base that is 2 feet wide and 10 feet long. The sides of the trough are also 2 feet wide by 10 feet long, and are to be placed so they make an angle θ with the vertical.



   (a) What is the area, in terms of θ, of a cross section of the trough perpendicular to its long side? What is the volume of the trough?

   (b) What angle θ will give the trough the largest volume, and what is that volume? [Hint: you can always replace $\cos^2(\theta)$ with $1 - \sin^2(\theta)$.]

5. (From the Winter, 2007 Math 115 Final Exam) Suppose that f and g are continuous functions with

$$\int_0^2 f(x)\,dx = 5 \qquad \text{and} \qquad \int_0^2 g(x)\,dx = 13.$$

Compute the following. If the computation cannot be made because something is missing, explain clearly what is missing.

   (a) $\int_4^6 f(x-4)\,dx$

   (b) $\int_{-2}^0 2g(-t)\,dt$

   (c) $\int_2^0 (f(y)+2)\,dy$

   (d) $\int_2^2 g(x)\,dx$

   (e) Suppose that f is an even function. Find the average value of f from -2 to 2.