Douglass Houghton Workshop, Section 2, Tue 11/21/23

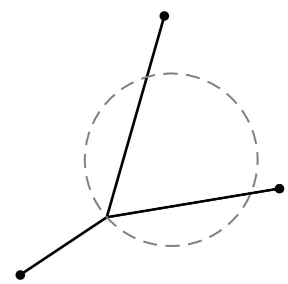
Worksheet Quoth the Raven, "Nevermore"

1. Shortest Network. Last time we used calculus to show that a Λ-shaped network can be improved if the vertex angle is less than 120°.

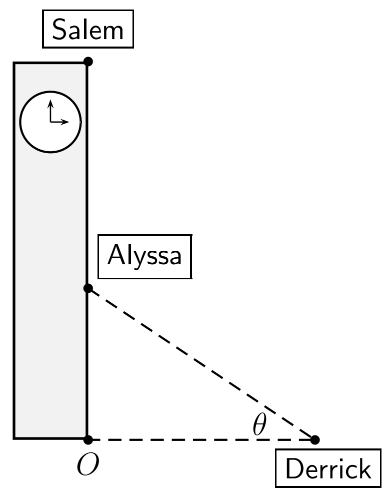
   (a) Prove that the network to the right is NOT minimizing. You don't need to find the optimal network, just prove that this one can be improved. Hint: consider the portion of the network that is inside the circle.

   (b) What allowed that trick to work? Phrase your answer like this: "Any network which contains ______ can be improved."

   (c) Put it all together, and explain where the soap puts the roundabout.



2. (Adapted from a Winter, 2005 Math 115 exam) One day Salem notices that the door to the Burton Tower carillon has been left open. They can't resist the urge to climb to the top of the tower and barricade themselves in. They then begin to serenade the campus with repeated trumpet solos. The University asks Alyssa to climb the tower and retake the carillon. Meanwhile on the ground, 30 feet from the tower, Derrick reads an epic poem that he composes on the spot, describing the day's events. He looks up at an angle θ to see Alyssa.



   (a) Find the rate of change of Alyssa's distance from the point O with respect to θ.

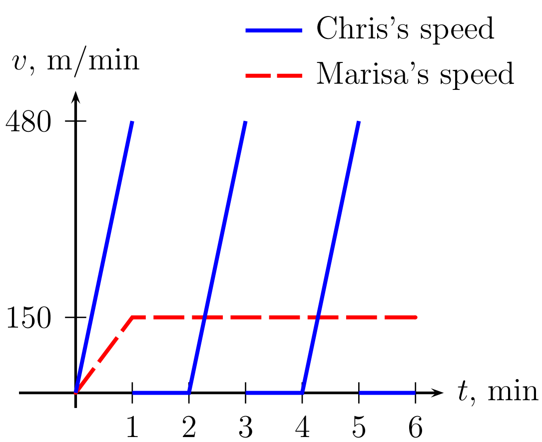
   (b) If the distance from point O to Salem is 200 ft and Alyssa climbs at a constant 8 ft/sec, what is the rate of change of θ with respect to time when Alyssa is halfway up?

   (c) When Alyssa is halfway up, Salem drops the end of a rope down to help her. The end of the rope falls with a constant acceleration of 32 ft/sec^2. When does Alyssa catch it, and what is its speed when she does?

   (d) Derrick watches the end of the rope as it drops, and also begins backing away from the tower at a rate of 5 ft/sec. How fast is the angle of his gaze changing when Alyssa catches the rope?

3. (Adapted from a Winter 2009 Math 115 Exam.) Marisa and Chris, after much friendly trash talk about who's the fastest runner, decide to have a race. The two employ very different approaches.

   - Marisa takes the first minute to accelerate to a slow and steady pace which she maintains through the remainder of the race.
   - Chris, on the other hand, spends the first minute accelerating to faster and faster speeds until he's exhausted and has to stop and rest for a minute—and then he repeats this process until the race is over. The graph below shows their speeds (in meters per minute), t minutes into the race. (Assume that the pattern shown continues for the duration of the race.)



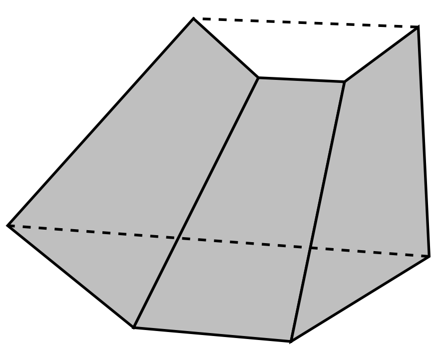
   (a) What is Marisa's average speed over the first two minutes of the race? What is Chris's?

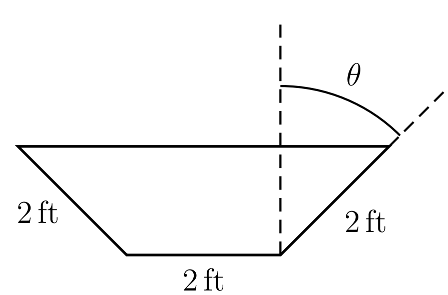
   (b) Chris immediately gets ahead of Marisa at the start of the race. How many minutes into the race does Marisa catch up to Chris for the first time?

   (c) Draw graphs of Chris's and Marisa's positions at time t. Be as precise as possible.

   (d) If the race is 720 meters total, who wins? What if it's 721 meters?

4. A trough, as shown below, is to be made with a base that is 2 feet wide and 10 feet long. The sides of the trough are also 2 feet wide by 10 feet long, and are to be placed so they make an angle θ with the vertical.





   (a) What is the area, in terms of θ, of a cross section of the trough perpendicular to its long side? What is the volume of the trough?

   (b) What angle θ will give the trough the largest volume, and what is that volume? [Hint: you can always replace $\cos^2(\theta)$ with $1 - \sin^2(\theta)$.]