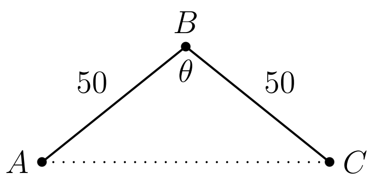
Douglass Houghton Workshop, Section 2, Thu 11/16/23

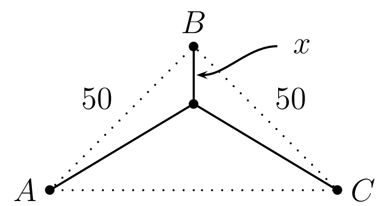
Worksheet Past is Prologue

1. Shortest Network. So far we've looked at the case where the cities are at the corners of an isosceles triangle like the one below. We know:



- When angle B is $70°$ or $90°$, it is possible to improve upon the Λ-shaped network shown by building a roundabout south of B and connecting it to all three cities.
- However, when B is $150°$, the Λ is better than all possible ⅄'s.

   (a) Suppose the measure of angle B is θ. Use the law of cosines to write a formula for the length of the ⅄-shaped network to the right, in terms of θ and x.



   (b) Call that function $L_\theta(x)$. Put your calculator in degrees mode and plot $L_{70}(x)$, $L_{90}(x)$, and $L_{150}(x)$, for x from 0 to 50. Put the graphs on the board.

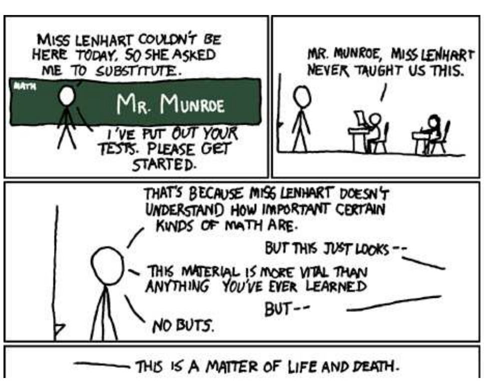
   (c) The shapes seem about the same. What can you see in the graphs that explains the fact that in two cases the Λ can be improved, and in the other it can't? (Remember the Λ is $x = 0$.)

   (d) Use calculus to figure out which Λ's can be improved, and which can't. State the result in the form: "Any Λ-shaped network with an angle smaller than ______ can be improved".

   (e) This is for those who like to compute and simplify. Show that the function $L_\theta(x)$ defined above is always concave up, by finding and simplifying its second derivative.

2. Suppose $\int_4^9 (4f(x) + 7)\,dx = 315$. Find $\int_4^9 f(x)\,dx$.

3.

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The velociraptor spots you 40 meters away and
attacks, accelerating at 4 m/s^2 up to its top
speed of 25 m/s.  When it spots you, you begin
to flee, quickly reaching your top speed of 6
m/s.  How far can you get before you're caught
and devoured?
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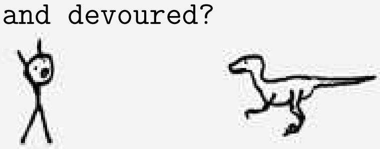
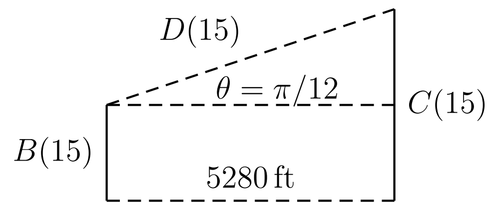


4. (Adapted from a Fall, 2011 Math 115 Final Exam problem) Briza takes the train to Indiana for Thanksgiving, where she plans to watch the Taylor Swift movie with friends.. At one point during the trip the tracks run parallel to a road, which is a mile away. The train is going quite slowly (6 ft/sec). Briza spots a Maserati sports car even with the train on the road, and turns her head as she watches it pull ahead. Let $C(t)$ be the distance between the car and its starting point, and $B(t)$ be Briza's distance from her starting point. After watching the car for 15 seconds, Briza has rotated her head $\pi/12$ radians.



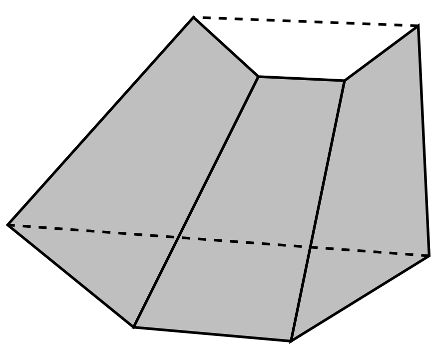
(a) Initially the car is 1 mile (5280 ft) due east of the train. Find the distance between Briza and the car 15 seconds after she starts watching it.

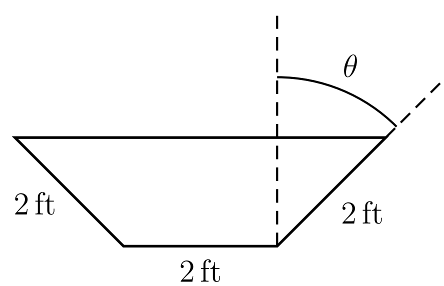
(b) Let $\theta(t)$ be the angle Briza has turned her head after tracking the car for t seconds. Write an equation for the distance between Briza and the car at time t. (Your answer may involve $\theta(t)$.)

(c) If at precisely 15 seconds, Briza is turning her head at a rate of .01 radians per second, what is the instantaneous rate of change of the distance between Briza and the car?

(d) What is the speed of the car at 15 seconds?

5. A trough, as shown below, is to be made with a base that is 2 feet wide and 10 feet long. The sides of the trough are also 2 feet wide by 10 feet long, and are to be placed so they make an angle θ with the vertical.





(a) What is the area, in terms of θ, of a cross section of the trough perpendicular to its long side? What is the volume of the trough?

(b) What angle θ will give the trough the largest volume, and what is that volume? [Hint: you can always replace $\cos^2(\theta)$ with $1 - \sin^2(\theta)$.]