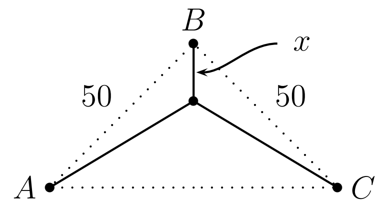
Douglass Houghton Workshop, Section 2, Tue 11/14/23

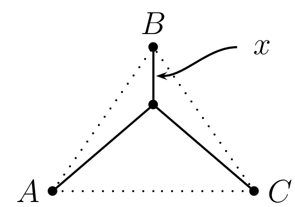
Worksheet Once More, Unto the Breach, Dear Friends

1. We've been working on the problem of finding the shortest road network between three cities in the plane.

   In the case we considered, the three cities were at the corners of a 45°-45°-90° triangle with legs 50 miles long. The simplest idea is to just build roads along the legs; that makes a network of length 100. But by constructing a ⅄-shaped network like the one at the right, we found

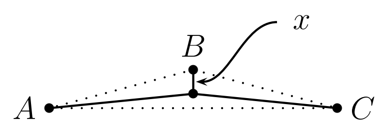
   

   - The length of the network is $x + 2\sqrt{2500 - 100x\cos(45) + x^2}$.
   - We can improve from the simple 2-road solution ($x = 0$, length $= 100$) by increasing x. For instance, when $x = 10$, the network has a length of about 97.

   (a) Consider the case where the triangle is still isosceles and the legs still have length 50, but the angle at B is 70°. Write a formula for the length of the network.

   (b) Can you find a value of x which beats the 2-road solution ($x = 0$, length $= 100$)?

   (c) Now suppose the vertex angle is very obtuse—say 150°. Find a formula for the length of the network.

   (d) Can you beat the 2-road solution in this case?

   (e) Suppose the vertex angle is θ. Write a formula for the length of the network.

2. November is National Novel Writing Month, and many people around the country attempt to complete a first draft of a novel in the course of the month. One of them is Mark's friend Chris. At the end of every day November she uploaded her manuscript to a website (`nanowrimo.org`), which counts how many words she has written. Here are her counts, rounded to the nearest hundred:

Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count
1	1700	4	7500	7	8700	10	9400	13	14800
2	3600	5	8300	8	8700	11	11800	14	17000
3	5800	6	8300	9	9400	12	13800		

   (a) Let x be the time in days since the start of November, and let $W(x)$ be the total number of words Chris has written at time x. Assume that each day Chris writes at a steady rate, from midnight to midnight. (But different rates for different days.) Draw a graph of $W(x)$ for the second week (x from 7 to 14).

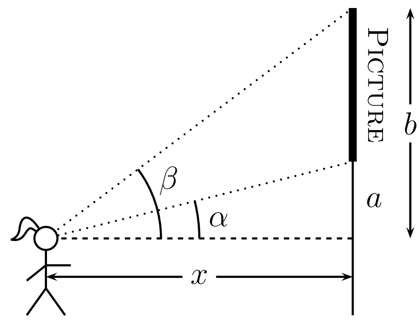
   (b) Let $w(x)$ be the derivative of $W(x)$. Draw a graph of $w(x)$ for x from 7 to 14.

   (c) Now consider the function $F(t)$, which is the area between the line $x = 7$, the line $x = t$, the x-axis, and the graph of $w(x)$. Make a table of values showing $F(7), F(8), \ldots, F(14)$. What do you notice? Explain this result.

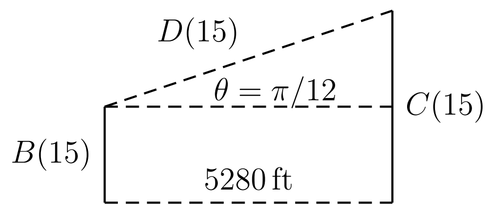
3. High on the wall at Faiza's home hangs a picture of Faiza and her twin brother Nabil, with all the other twins in her family in the background. Its bottom is a feet above eye level, and its top is b feet above eye level. If you stand far away from the wall, you can't see the picture well. But if you stand close to the wall, you can't see well either! So the question is: how far from the wall should you stand in order to have the best view?

   (a) Let α and β be the angles between eye level and the bottom and top of the picture, as shown. x is your distance from the wall. Find α and β in terms of x, a, and b.

   (b) $\beta - \alpha$ is the angle that the picture takes up in your field of vision. So find the value of x that maximizes $\beta - \alpha$.



4. (Adapted from a Fall, 2011 Math 115 Final Exam problem) Briza takes the train to Indiana for Thanksgiving, where she plans to watch the Taylor Swift movie with friends.. At one point during the trip the tracks run parallel to a road, which is a mile away. The train is going quite slowly (6 ft/sec). Briza spots a Maserati sports car even with the train on the road, and turns her head as she watches it pull ahead. Let $C(t)$ be the distance between the car and its starting point, and $B(t)$ be Briza's distance from her starting point. After watching the car for 15 seconds, Briza has rotated her head $\pi/12$ radians.



   (a) Initially the car is 1 mile (5280 ft) due east of the train. Find the distance between Briza and the car 15 seconds after she starts watching it.

   (b) Let $\theta(t)$ be the angle Briza has turned her head after tracking the car for t seconds. Write an equation for the distance between Briza and the car at time t. (Your answer may involve $\theta(t)$.)

   (c) If at precisely 15 seconds, Briza is turning her head at a rate of .01 radians per second, what is the instantaneous rate of change of the distance between Briza and the car?

   (d) What is the speed of the car at 15 seconds?