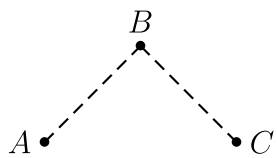
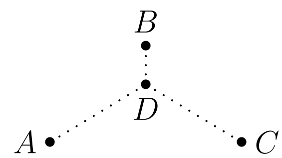
Douglass Houghton Workshop, Section 2, Thu 11/09/23

Worksheet Now is the Winter of our Discontent

1. The three cities in the pictures below are at the corners of a 45°-45°-90° triangle whose legs are 50 miles long. The three mayors, working together, would like to build roads between them in such a way that there is a way to get from any one city to any other city.





(Say, A is Ann Arbor, B is Flint, and C is Port Huron.) The first, simple proposal (on the left) is to build a road from A to B and another from B to C. That would certainly work. But roads are expensive, and one of the mayors (who, luckily, studied calculus) proposes building roads from A and C to a point D just south of B, then building a road north from there to B.

   (a) Let x be the length of the north-south road in the second proposal. What does it mean if $x = 0$?

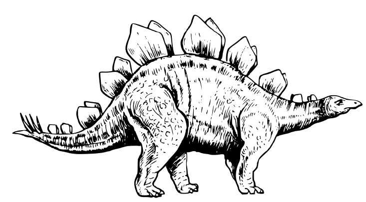
   (b) Calculate the total length of the new network in terms of x. Hint: "Law of cosines".

   (c) Can you find a value of x which will produce a shorter network than the simple proposal?

2. (Adapted from a Fall, 2001 Math 115 final exam) Rachel starts a business selling green ceramic models of dinosaurs. The best-seller is a stegasaurus.

Her cost and revenue functions are

$$C(q) = 400 + 8q \qquad \text{and} \qquad R(q) = 60q^{.75}$$



where q is the number of stegosauruses produced.

   (a) What is the product's fixed cost?

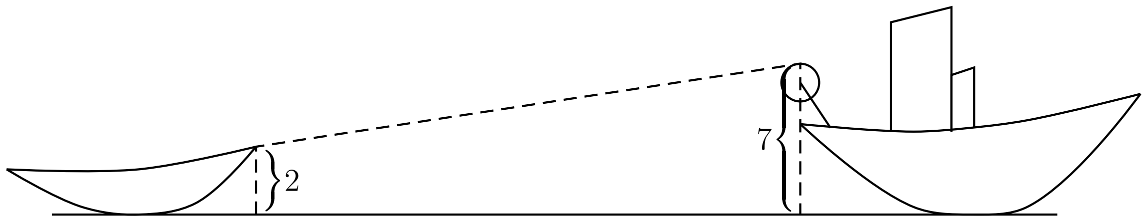
   (b) Last year, Rachel produced 2400 stegosauruses. What was her profit?

   (c) Find formulas for the marginal cost and marginal revenue, and evaluate at $q = 2400$.

   (d) Rachel would like to increase production and do better this year. Based on the marginal cost and marginal revenue *at this point* ($q = 2400$), explain whether her strategy is sound.

3. (Adapted from a Fall, 2006 Math 115 Final Exam.) Ashley is swimming the first leg of a triathalon off the coast of Hawaii, when she gets a cramp and is picked up by a rescue boat. Unfortunately, the rescue boat later breaks down, and requires a tow.

   The tugboat captain throws Ashley a line, and she attaches it to her boat 2 meters above the water line. The other end of the cable is attached to a wheel of radius 0.5 meters sitting on the back of the tugboat. The top of the wheel is 7 meters above the water, and turns at a constant rate of 1 revolution per second. [See the figure below—not drawn to scale.]



   (a) At what rate is the length of the cable between the two boats changing?

   (b) How fast is Ashley's boat being pulled forward when it is 10 meters away from the tugboat?

4. November is National Novel Writing Month, and many people around the country attempt to complete a first draft of a novel in the course of the month. One of them is Mark's friend Chris. At the end of every day November she uploaded her manuscript to a website (`nanowrimo.org`), which counts how many words she has written. Here are her counts, rounded to the nearest hundred:

Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count
1	1700	4	7500	7	8700	10	9400	13	14800
2	3600	5	8300	8	8700	11	11800	14	17000
3	5800	6	8300	9	9400	12	13800		

   (a) Let x be the time in days since the start of November, and let $W(x)$ be the total number of words Chris has written at time x. Assume that each day Chris writes at a steady rate, from midnight to midnight. (But different rates for different days.) Draw a graph of $W(x)$ for the second week (x from 7 to 14).

   (b) Let $w(x)$ be the derivative of $W(x)$. Draw a graph of $w(x)$ for x from 7 to 14.

   (c) Now consider the function $F(t)$, which is the area between the line $x = 7$, the line $x = t$, the x-axis, and the graph of $w(x)$. Make a table of values showing $F(7), F(8), \ldots, F(14)$. What do you notice? Explain this result.