Douglass Houghton Workshop, Section 2, Tue 10/24/23

Worksheet Magnificent

1. (This problem appeared on the Fall, 2008 Math 115 Final Exam) At the Michigan-Ohio State basketball game this year, the Michigan Band discovers that the amount of time it spends playing "Hail to the Victors" has a direct impact on the number of points our team scores. If the band plays for x minutes, then the Wolverines will score

$$W(x) = -.48x^2 + 7.2x + 63$$

points. Assume that the band can play for a maximum of 10 minutes.

   (a) How long should the band play to maximize the number of points Michigan scores?

   (b) The band affects how many points Ohio State scores as well. x minutes of playing results in the Buckeyes scoring

$$B(x) = -x^2 + 8x + 84$$

   points. Find the number of minutes the band should play to maximize the margin of victory for Michigan.

   (c) What will be the score of the game for the case you found in part (b)?

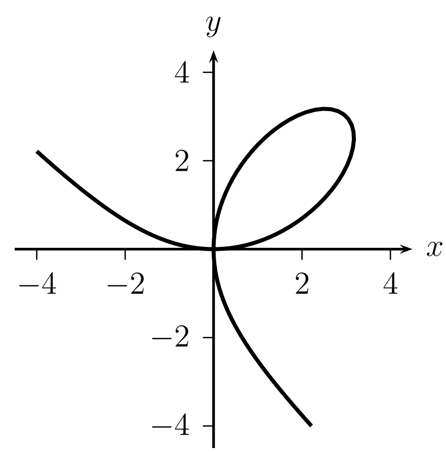
2. (This problem appeared on a Winter, 2005 Math 115 Exam) An example of Descartes' folium, shown in the picture to the right, is given by $x^3 + y^3 = 6xy$.

   (a) Show that the point $(3, 3)$ is on the graph.

   (b) Find the equation of the tangent to the graph at the point $(3, 3)$.

   (c) For what value(s) of x will the tangent to this curve be horizontal? [You do not need to solve for both x and y—just show y in terms of x.]

   (d) (Added for DHSP) Oh heck, go ahead and find the point(s).



3. Gabriella drives on the Pennsylvania Turnpike for Thanksgiving. She enters at Lawrence, where a camera takes a picture of the car's license plate. The picture is processed and the licensce plate stored in a database, along with the time the picture was taken. Later Gabriella exits at Neshaminy Falls, where another picture is taken. A week afterward, she receives a bill in the mail along with a speeding citation, stating that she was going exactly 75 mph at some point on her trip. How does the Mean Value Theorem allow the authorities to be sure that happened?

4. Last time we investigated rules for how a population of Eurasian Otters might change. Let's nail down the essential features of all similar rules. Here's what we know:

Rule	Equilibrium	Stable?
$P(n+1) = 1.5P(n) - 200$	400	No
$P(n+1) = .75P(n) + 125$	500	Yes

An **equilibrium** is a population that will stay constant from year to year. An equilibrium $\hat{P}$ is **stable** if when the population starts a little above or below $\hat{P}$, it moves toward $\hat{P}$. Otherwise $\hat{P}$ is **unstable**.

(a) Add rows to the table for these rules. You can reason either numerically, graphically, algebraically, or with words.

$P(n+1) = .4P(n) + 600$ $\quad$ $P(n+1) = -1.3P(n) + 460$

$P(n+1) = 1.1P(n) - 330$ $\quad$ $P(n+1) = P(n) + 300$

$P(n+1) = -.5P(n) + 1200$ $\quad$ $P(n+1) = -P(n) + 300$

(b) Now do $P(n+1) = mP(n) + b$, where m and b are constants.

5. Suppose $h(x)$ is a continuous function defined for all real numbers x. The derivative and second derivative of $h(x)$ are given by

$$h'(x) = \frac{2x}{3(x^2-1)^{2/3}} \qquad \text{and} \qquad h''(x) = -\frac{2(x^2+3)}{9(x^2-1)^{5/3}}.$$

(a) Find the all critical points and local extrema of $h(x)$. Use calculus to classify the critical points and justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema.

(b) Find all inflection points of $h(x)$, and justify that you've found them all.

6. Suppose Gabby is walking along the shore of Lake Michigan with her dog Leo. Gabby throws a ball 30 meters down the beach and 16 meters out into the water.

Leo, being practical, wants to get to the ball as quickly as possible. The thing is that he can run faster than he can swim; his running speed on the beach is 9 meters per second, and he can swim 3 meters per second. How should Leo (who has an intuitive notion of calculus) get to the ball?