Worksheet Labradoodle

1. In "The 12 days of Christmas", a certain poultry-afficianado receives a number of gifts from her true love:

   Day 1: A partridge in a pear tree. How to get it down?

   Day 2: 2 turtle doves, and another partridge in a pear tree. Is it the same tree?

   Day 3: 3 French hens, 2 more turtle doves, and another partidge.

   . . .

   Day 12: 12 drummers drumming (loudly), eleven pipers piping (make them stop!), . . ., and yet another partridge in a pear tree.

   (a) How many total partridges does the heroine receive, over the course of the song? How many turtle doves?

   (b) If item 1 is "partridge", item 2 is "turtle dove", etc., then write a formula for the total number of item n's received.

   (c) Of which item does Mr. Truelove send the most? (Solve using calculus.)

2. Scarlett is studying a population of Eurasian Otters in Germany. Suppose that the population changes according to the rule:
   $$P(n+1) = 1.5P(n) - 200$$
   where $P(0)$ is the population in 2023, $P(1)$ is the population 1 year later, etc. (P is measured in otters.)

   (a) Make up a (short) story about Eurasian Otters that yields that formula as the result.

   (b) Suppose $P = 320$ in 2023. What will happen in the long run?

   (c) Suppose instead that $P = 440$ in 2023. Now what happens?

   (d) A population is in **equilibrium** if it stays the same from year to year. Is there an equilibrium number for this population?

   (e) Explain these results pictorially by drawing the graphs of $y = x$ and $y = 1.5x - 200$. Start at $(320, 320)$, go down to the other graph, and then over to $y = x$. That's the new population. Repeat. Then start at 440.

3. Repeat the last problem, but for the rule
   $$P(n+1) = .75P(n) + 125.$$

4. (This problem appeared on a Winter, 2005 Math 115 Exam) An example of Descartes' folium, shown in the picture to the right, is given by $x^3 + y^3 = 6xy$.
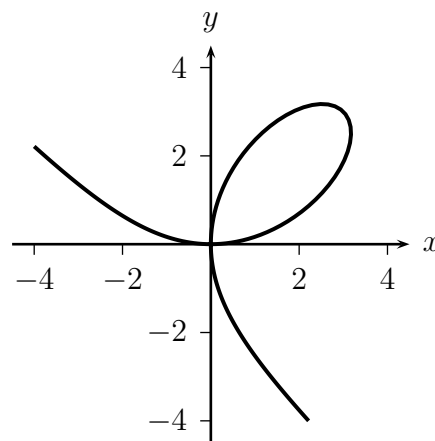
   (a) Show that the point $(3, 3)$ is on the graph.

   (b) Find the equation of the tangent to the graph at the point $(3, 3)$.

   (c) For what value(s) of x will the tangent to this curve be horizontal? [You do not need to solve for both x and y—just show y in terms of x.]

   (d) (Added for DHSP) Oh heck, go ahead and find the point(s).

5. (This problem appeared on the Fall, 2008 Math 115 Final Exam) At the Michigan-Ohio State basketball game this year, the Michigan Band discovers that the amount of time it spends playing "Hail to the Victors" has a direct impact on the number of points our team scores. If the band plays for x minutes, then the Wolverines will score

$$W(x) = -.48x^2 + 7.2x + 63$$

points. Assume that the band can play for a maximum of 10 minutes.

   (a) How long should the band play to maximize the number of points Michigan scores?

   (b) The band affects how many points Ohio State scores as well. x minutes of playing results in the Buckeyes scoring

$$B(x) = -x^2 + 8x + 84$$

points. Find the number of minutes the band should play to maximize the margin of victory for Michigan.

   (c) What will be the score of the game for the case you found in part (b)?

6. (This problem appeared on a Winter, 2004 Math 115 exam. Really!) While exploring an exotic spring break location, you discover a colony of geese who lay golden eggs. You bring 20 geese back with you. Suppose each goose can lay 294 golden eggs per year. You decide maybe 20 geese isn't enough, so you consider getting some more of these magical creatures. However, for each extra goose you bring home there are less resources for all the geese. Therefore, for each new goose the amount of eggs produced will decrease by 7 eggs per goose per year. How many more geese should you bring back if you want to maximize the number of golden eggs per year laid?

7. (An old team homework problem.) Let $f(x) = x^2 - 2x + 13$ and $g(x) = -x^2 - 2x - 5$.

   (a) Draw $y = f(x)$ and $y = g(x)$ on the same set of axes. How many lines are tangent to both graphs?

   (b) Find the equations of those lines.