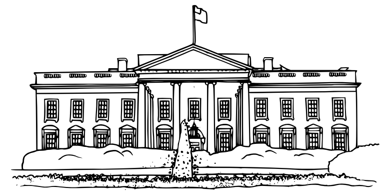
Douglass Houghton Workshop, Section 2, Thu 10/12/23

Worksheet Koala



1. We still have this $1/z$ scale model of the White House, which we plan on blowing up. We want to decide what speed to run the film at, so that when we slow it down to 24 frames per second, we get a realistic explosion.

   (a) Last time we showed that an object will fall $16t^2$ feet in t seconds. So how long does it take for an object to fall off the real white house, which is H feet tall? How many frames will that be, if we film it at 24 frames per second and show it at the same speed?

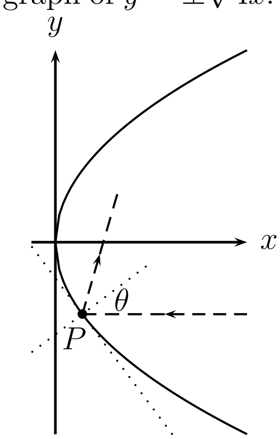
   (b) How long does it take an object to fall off the top of the model?

   (c) How many frames per second should you film to get the right number of frames to make it look like the model is full-sized?

2. Last time we thought about a parabolic mirror in the shape of the graph of $y = \pm\sqrt{4x}$.

   So far we've found:

   - A light ray $y = -b$ hits the mirror at $P = (b^2/4, -b)$.
   - The slope of the tangent at that point is $-2/b$.
   - The normal line at the same point has slope $b/2$.
   - When a line makes an angle θ with the x-axis, it has slope $\tan\theta$.
   - So if we call the angle between the normal line and the horizontal θ, then $\theta = \tan^{-1}(b/2)$.



   (a) Draw the picture on the board.

   (b) To the ray, the mirror looks flat, just like the tangent line. Draw the reflected ray. What angle does it make with the x-axis?

   (c) We know that $\sin 2x = 2\sin x\cos x$ and $\cos 2x = \cos^2 x - \sin^2 x$. Use those to find a formula for $\tan 2x$ in terms of $\tan x$.

   (d) What is the slope of the reflected ray?

   (e) Write an equation for the reflected ray.

   (f) Where does the reflected ray intersect the x-axis? What is surprising about this answer?

   (g) Graph several rays, with their reflections.

   (h) What's cool about this type of mirror?

3. Gabriella drives on the Pennsylvania Turnpike for Thanksgiving. She enters at Lawrence, where a camera takes a picture of the car's license plate. The picture is processed and the licensce plate stored in a database, along with the time the picture was taken. Later Gabriella exits at Neshaminy Falls, where another picture is taken. A week afterward, she receives a bill in the mail along with a speeding citation, stating that she was going exactly 75 mph at some point on her trip. How does the Mean Value Theorem allow the authorities to be sure that happened?

4. (An old team homework problem.) Let $f(x) = x^2 - 2x + 13$ and $g(x) = -x^2 - 2x - 5$.

   (a) Draw $y = f(x)$ and $y = g(x)$ on the same set of axes. How many lines are tangent to both graphs?

   (b) Find the equations of those lines.

5. Let $f^{(n)}(x)$ denote the nth derivative of f. If $f(x) = e^{-2x}$, find $f^{(531)}(x)$. Is $f^{(531)}(x)$ increasing or decreasing? Concave up or concave down? Try graphing $f^{(531)}$ without your calculator, then check with the calculator.

6. Molecules absorb far-infrared radiation because its excites their rotation. The absorption coefficient a of a given liquid varies with the frequency ω of the radiation according to

$$a(\omega) = \frac{10}{\omega^2 - 2c\omega + 125}$$

   where c is some constant $(0 \le c \le 11)$.

   (a) For what value of the frequency ω is the absorption a maximum?

   (b) Graph $a(\omega)$ for $c = 11$. How would you describe the shape of this graph?

   [Note: with appropriate parameters this function describes the shapes of the lines in many kinds of spectroscopy].

7. (This problem appeared on a Winter, 2004 Math 115 exam. Really!) While exploring an exotic spring break location, you discover a colony of geese who lay golden eggs. You bring 20 geese back with you. Suppose each goose can lay 294 golden eggs per year. You decide maybe 20 geese isn't enough, so you consider getting some more of these magical creatures. However, for each extra goose you bring home there are less resources for all the geese. Therefore, for each new goose the amount of eggs produced will decrease by 7 eggs per goose per year. How many more geese should you bring back if you want to maximize the number of golden eggs per year laid?