Douglass Houghton Workshop, Section 2, Tue 10/10/23

Worksheet Journey into Night

1. As we know, Sofia spent much of her young life walking through the forest with her microscope, observing nature. One day, she is wandering through the woods and comes to the top of a cliff. For fun, she drops a rock over the edge.

   (a) The rock's downward acceleration due to gravity is $-32\,\mathrm{ft/sec}^2$. So what is its velocity t seconds after it is dropped? (Keep in mind that the derivative of velocity is acceleration, and the initial velocity is 0.)

   (b) How far has the rock fallen after t seconds?

   (c) Suppose the rock takes 3.5 seconds to hit the bottom. How high is the cliff?

   (d) Explain a general method for finding the height of any cliff.

2. Suppose you construct a $1/z$ scale model of the White House, in order to film it blowing up. You will show the film at 24 frames per second. How many frames per second should you *film* so that when you slow the speed down, things will fall at believable speeds?

3. Consider a mirror in the shape of the graph of $y = \pm\sqrt{4x}$.

   (a) Draw the mirror (make it big). What shape is it?

   (b) Draw a light ray travelling leftward along the line $y = -b$, where b is some positive number (making $-b$ negative). At what point P does the ray hit the mirror?

   (c) Find, in terms of b, the slope of the tangent to the mirror at P.

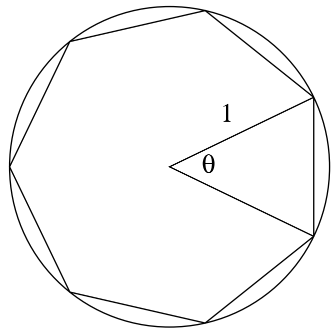
   (d) The *normal* to a curve at a point is the line through that point which is perpendicular to the tangent line. Find the slope of the normal to the mirror at P, and draw both the normal and tangent lines on your graph.

   (e) Suppose a line makes an angle θ with the positive x-axis. What is the slope of the line?

   (f) Let θ be the angle the normal to the mirror at P makes with the light ray $y = -b$. Can you write θ in terms of b? Hint: Use (**??**) and (**??**).

   To be continued...

4. (This problem explains why $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$, but only when θ is measured in radians.) Consider a regular n-sided polygon inscribed in a circle of radius 1.



(a) Let A_n be the area of the polygon. What does A_n approach as n gets large? $\lim_{n\to\infty} A_n = \boxed{}$

(b) We can compute A_n by dividing the polygon up into triangles which have a vertex at the center. Let θ be the vertex angle (in radians). What is θ in terms of n?

(c) What happens to θ as n gets large?

(d) What is the area of one of the triangles, in terms of θ?

(e) What is A_n in terms of θ?

(f) Substitute into the equation from part (a) so that it includes θ's but not n's. Simplify it as much as you can. Hint: $\sin(2x) = 2\sin(x)\cos(x)$.

(g) What would change if we measured θ in degrees instead of radians?

5. (From Fall, 2011, Math 115 Exam 2) Let $f(x) = \ln(x)$. Use the table of values below for $g(x)$ and $g'(x)$ to answer the following questions.

(a) If $F(x) = f(g(x))$, find $F'(4)$.

(b) If $G(x) = g^{-1}(x)$, find $G'(4)$.

(c) If $H(x) = \tan(g(x))$, find $H'(3)$.

(d) If $E(x) = e^{f(x)g(x)}$, find $E'(2)$.

x	2	3	4
$g(x)$	1	4	6
$g'(x)$	5	3	2

6. (An old team homework problem.) Let $f(x) = x^2 - 2x + 13$ and $g(x) = -x^2 - 2x - 5$.

(a) Draw $y = f(x)$ and $y = g(x)$ on the same set of axes. How many lines are tangent to both graphs?

(b) Find the equations of those lines.

7. (This problem appeared on a Winter 2007 Math 115 exam) Suppose f and g are differentiable functions with values given by the table below.

(a) If $h(x) = f(x)g(x)$, find $h'(3)$.

(b) If $j(x) = \dfrac{(g(x))^3}{f(x)}$, find $j'(1)$.

(c) If $d(x) = x\ln\left(e^{f(x)}\right)$, find $d'(3)$.

(d) If $t(x) = \cos(g(x))$, find $t'(1)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	9	-3	7
3	4	11	15	-19