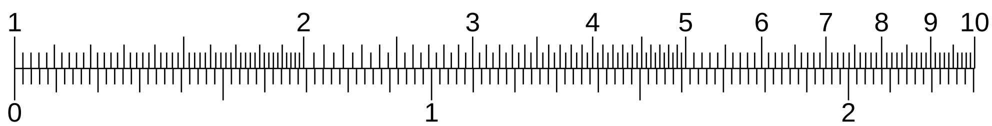
Douglass Houghton Workshop, Section 2, Tue 09/19/23

Worksheet Go Forth and Multiply

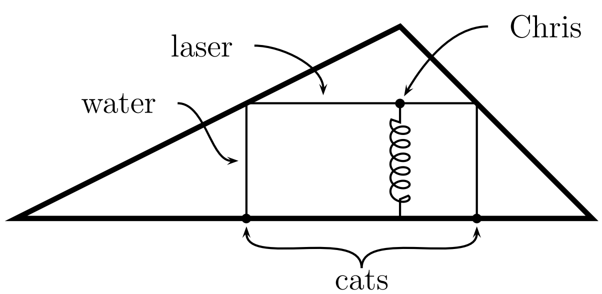
1. Explain how to use two rulers to add numbers.

2. Explain how a slide rule is able to multiply two numbers. This picture may be helpful:



3. Why is it necessary to define the derivative in terms of a limit? Draw a picture that describes how the derivative is the limit of the slopes of some lines.

4. Chris is being raised on a giant spring-loaded platform right under the peak of a triangular room. Laser beams are being emitted from his head, parallel to the ground, until they hit the walls. Where they hit the walls, drops of water fall down, then land in the mouths of two cats. As Chris goes up, the cats follow the drops toward the base of the platform.



   (a) Let $h(t)$ be Chris's height at time t, and let $w(t)$ be the distance between the two cats. Are they continuous functions? Is $h(t) - w(t)$ a continuous function?

   (b) When t is close to 0 (so Chris's head has just come through the floor), what can you say about $h(t) - w(t)$?

   (c) Later on, when Chris is near the end of his journey and about to hit the top, what can you say about $h(t) - w(t)$?

   (d) Use the Intermediate Value Theorem to show that at some time the distance between the cats is the same as Chris's height off the floor.

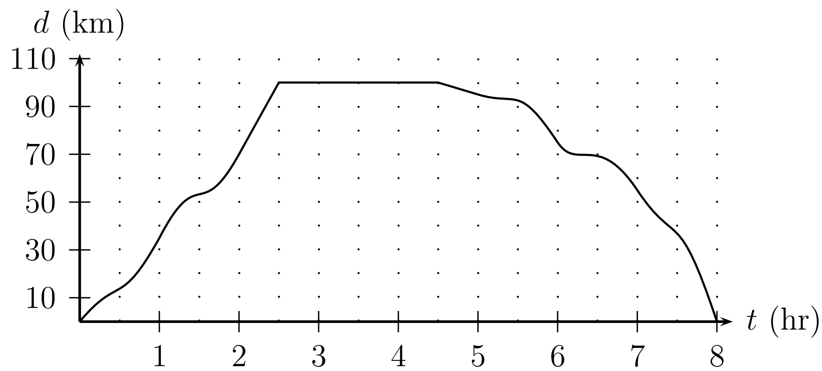
5. Use the definition of the derivative to find $f'(x)$ when $f(x) = \sqrt{x}$. Hint: $(a-b)(a+b) = a^2 - b^2$.

6. (This problem appeared on a Winter, 2014 Math 115 Exam.) The air in a factory is being filtered so that the quantity of a pollutant, P (in mg/liter), is decreasing exponentially. Suppose t is the time in hours since the factory began filtering the air. Also assume 20% of the pollutant is removed in the first five hours.

   (a) What percentage of the pollutant is left after 10 hours?

   (b) How long is it before the pollution is reduced by 50%?

7. (This problem appeared on a Winter, 2014 Math 115 Exam) A ship's captain is making a round trip voyage between two ports. The ship sets sail from Port Jackson at noon, arrives at Port Kembla some time later, waits there for a while, and then returns to Port Jackson. Let $s(t)$ be the ship's distance, in kilometers, from its starting point of Port Jackson, t hours after noon. A graph of $d = s(t)$ is shown below.



   (a) How far is Port Kembla from Port Jackson?

   (b) How long does the ship wait in Port Kembla?

   (c) What is the ship's average speed during the return trip?

   (d) Estimate the ship's instantaneous velocity at 1pm.

   (e) Sometime after 5pm, there is a time when the ship's instantaneous velocity is 0 km/hr. When does this occur?