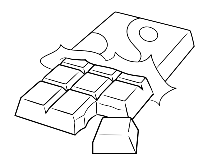
Douglass Houghton Workshop, Section 2, Thu 09/14/23

Worksheet Fluffernutter

1. Alex has noticed that her tastes changed over the last year. A year ago she spent about 15 hours a week managing a chocolate shop, and 10 hours making spotify playlists for her favorite literary characters. Gradually school took over her life, and though there have been some ups and downs in her schedule, the general trend is that she's spent less time per week on both. Now, 52 weeks later, she spends only 3 hours a week managing a chocolate shop and 5 hours a week making spotify playlists for her favorite literary characters.

   Let $C(t)$ be the number of hours Alex spent managing a chocolate shop in week t, and let $S(t)$ be the number of hours she spent making spotify playlists for her favorite literary characters. Assume $C(t)$ and $S(t)$ are continuous functions of time.



   (a) What does it mean for a function to be continuous?

   (b) Are $C(t) + S(t)$, $C(t) - S(t)$, and $C(t)S(t)$ continuous?

   (c) Use one of the functions in part (a) together with the Intermediate Value Theorem to show that at some time in the last year, Alex was spending the same amount of time managing a chocolate shop and making spotify playlists for her favorite literary characters.

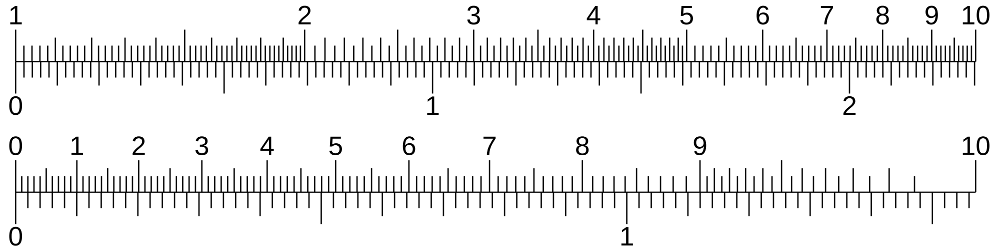
2. (This problem is adapted from a Fall, 2015 Math 115 Exam) Becky is jumping rope while Alyssa runs a stopwatch. There is a piece of tape around the middle of the rope. When the rope is at its lowest, the piece of tape is 2 inches above the ground, and when the rope is at its highest, the piece of tape is 68 inches above the ground. The rope makes two complete revolutions every second. When Alyssa starts her stopwatch, the piece of tape is halfway between its highest and lowest points and moving downward. The height H (in inches above the ground) of the piece of tape can be modeled by a sinusoidal function $C(t)$, where t is the number of seconds displayed on Alyssa's stopwatch.

   (a) Sketch a well-labeled graph of two periods of $C(t)$ beginning at $t = 0$.

   (b) Find a formula for $C(t)$.

   (c) Now Alyssa takes a turn at jumping. Becky resets the stopwatch and starts it over again. Let $G(w)$ be the height (in inches above the ground) of the piece of tape when Becky's stopwatch says w seconds. A formula for $G(w)$ is $G(w) = 41 + 38\cos(2\pi w)$. Becky is 60 inches tall. For how long (in seconds) during each revolution of the rope is the piece of tape higher than the top of Becky's head? (Assume Becky is standing straight while watching the stopwatch.)

3. What's the deal with these pictures? What are they good for?



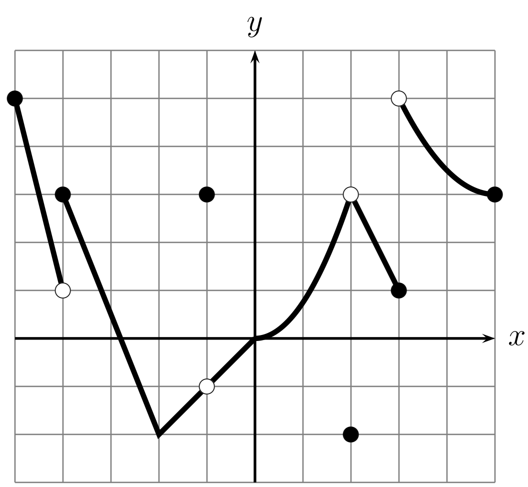
4. We've all seen 6-sided dice, and we presume they are "fair", in the sense that all 6 sides are equally likely to land on the bottom. Can you construct a fair 4-sided die? How about an 8-sided die? What other sizes are possible?

5. (This problem appeared on a Winter, 2013 Math 115 Exam.) The air in a factory is being filtered so that the quantity of a pollutant, P (in mg/liter), is decreasing exponentially. Suppose t is the time in hours since the factory began filtering the air. Also assume 20% of the pollutant is removed in the first five hours.

   (a) What percentage of the pollutant is left after 10 hours?

   (b) How long is it before the pollution is reduced by 50%?

6. (From a Fall, 2017 Math 115 Exam.) The graph of $y = Q(x)$ is shown. The gridlines are one unit apart.



   (a) On which of the following intervals is $Q(x)$ invertible?

   $[-4,-1]$ $[-2,3]$ $[2,5]$ $[-2,2]$

   (b) Find the following limits. Write "NI" if there you don't have enough information and "DNE" if the limit doesn't exist.

   i. $\lim_{x\to -1} Q(x)$ ii. $\lim_{w\to 2} Q(Q(w))$

   iii. $\lim_{h\to 0} \frac{Q(-3+h)-Q(-3)}{h}$ iv. $\lim_{x\to\infty} Q\left(\frac{1}{x}+3\right)$ v. $\lim_{x\to\frac{1}{3}} xQ(3x-5)$

   (c) For which values of $-5 < x < 5$ is the function $Q(x)$ <u>not</u> continuous?

   (d) For which values of $-5 < p < 5$ is $\lim_{x\to p^-} Q(x) \neq Q(p)$?