Douglass Houghton Workshop, Section 2, Thu 09/07/23

Worksheet Down the Rabbit Hole

1. *The Saga of Michael Phelps: Conclusion* Last time we found that Michael Phelps can always make himself dryer by splitting his towel, but there's apparently a limit to how dry he can get. In particular, here are the numbers for not splitting the towel at all and for splitting it into 10,000 pieces:

Towel Size	.25	.5	1	2	3	4
wetness (1 piece)	0.8000	0.6667	0.5000	0.3333	0.2500	0.2000
wetness (10,000 pieces)	0.7788	0.6065	0.3679	0.1354	0.0498	0.0183

Cutting into more than 10,000 pieces doesn't seem to make much difference. So for each towel T, there is a wetness $N(T)$ after normal toweling, and there seems to be a "magic number" $M(T)$, which is the *limit* to how dry Michael can get by splitting the towel.

(a) Make a graph with towel size on the x-axis and wetness on the y-axis. Plot the points you have for $N(T)$, the result of normal toweling, and $M(T)$, the result of split towelling.

(b) What's the formula for $N(T)$? (We found this previously).

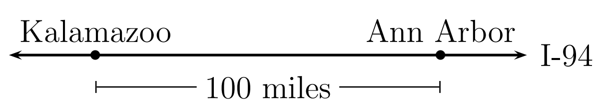
(c) What kind of function does $M(T)$ look like? Hint: Compare $M(1)$ with $M(2)$.

(d) Verify your guess by finding a formula that fits the data.

(e) Using the formula we found last time for splitting the towel into n parts, write a limit equation to express the result in part (d).

2. Kalamazoo is 100 miles west of Ann Arbor along Route 94.

Let $T(x)$ be the temperature in Fahrenheit at a point x miles west of Ann Arbor.

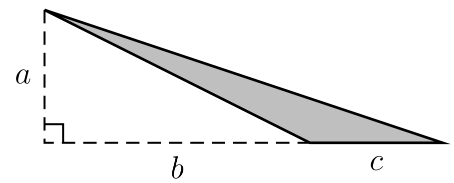


(a) Define a function A in terms of T so that $A(m)$ is the temperature in Fahrenheit at a point m miles east of Kalamazoo.

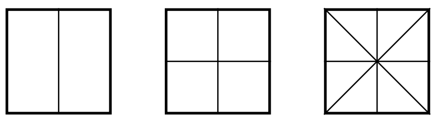
(b) Define a function B in terms of T so that $B(k)$ is the temperature in Fahrenheit at a point k **kilometers** east of Kalamazoo. (1 mile = 1.6 kilometers.)

(c) Define a function C in terms of T so that $C(k)$ is the temperature in **Celcius** at a point k kilometers east of Kalamazoo.

3. dBase™ was a database management system popular on IBM PCs back in the 80s, and still used in some places today. It included a programming language with limited capabilities; for instance, there was no command in the language to take the square root of a number. There were, however, two functions called `LOG`(x) and `EXP`(x) which produced $\ln(x)$ and e^x, respectively. How could you use them to produce $\sqrt{x}$?

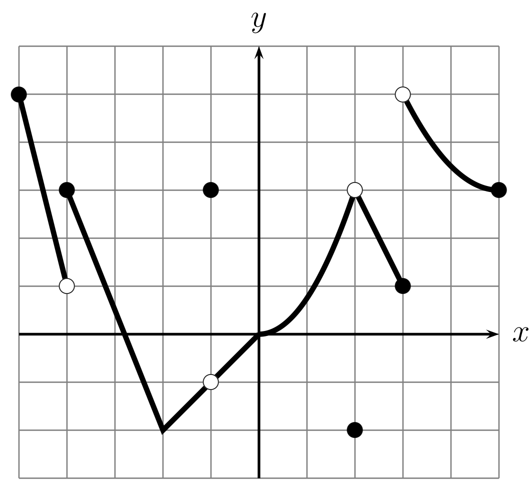
4. Find the area of the shaded triangle: 

5. Suppose you bake a square cake, 10 inches on a side and 2 inches high. You frost it on the top and all four sides (but not the bottom). We want to split the cake among n people, and we want everyone to get equal shares of cake and frosting. Last time we figured out how to do it for $n = 2$, $n = 4$, and $n = 8$:



We had a number of other ideas too. What other numbers of people can you accommodate? Explain exactly how to cut the cake and why it is fair.

6. (From a Fall, 2017 Math 115 Exam.) The graph of $y = Q(x)$ is shown. The gridlines are one unit apart.



(a) On which of the following intervals is $Q(x)$ invertible?

$$[-4, -1] \quad [-2, 3] \quad [2, 5] \quad [-2, 2]$$

(b) Find the following limits. Write "NI" if there you don't have enough information and "DNE" if the limit doesn't exist.

i. $\lim\limits_{x\to -1} Q(x)$ ii. $\lim\limits_{w\to 2} Q(Q(w))$

iii. $\lim\limits_{h\to 0} \dfrac{Q(-3+h) - Q(-3)}{h}$ iv. $\lim\limits_{x\to\infty} Q\left(\dfrac{1}{x} + 3\right)$ v. $\lim\limits_{x\to\frac{1}{3}} xQ(3x-5)$

(c) For which values of $-5 < x < 5$ is the function $Q(x)$ <u>not</u> continuous?

(d) For which values of $-5 < p < 5$ is $\lim\limits_{x\to p^-} Q(x) \neq Q(p)$?

7. (This problem appeared on a Winter, 2013 Math 115 Exam.) The air in a factory is being filtered so that the quantity of a pollutant, P (in mg/liter), is decreasing exponentially. Suppose t is the time in hours since the factory began filtering the air. Also assume 20% of the pollutant is removed in the first five hours.

(a) What percentage of the pollutant is left after 10 hours?

(b) How long is it before the pollution is reduced by 50%?