Douglass Houghton Workshop, Section 2, Tue 09/05/23

Worksheet Creamy Ginger Frosting

1. As we know, Marisa is obsessed with the color pink. She has pink slippers, pink gloves, pink pens, a pink calculator, and many more pink items. Currently she has 40 pink items. She'd really rather leave the color pink behind, but friends and family keep giving her new pink items every year.

   Write formulas for the number of pink items Marisa will have t years from now, under the following conditions:

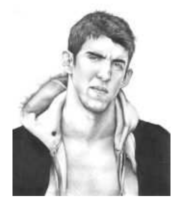
   (a) Marisa receives 5 new pink items every year.

   (b) In year t Marisa receives one new pink item for each two pink items she had in year $t-1$.

   (c) Marisa receives 1 pink item next year, 2 the year after that, 3 the year after that, etc.

2. Last time we found formulas for Michael Phelps' dampness after regular towelling and split towelling. Assuming Michael's body is $1\,\mathrm{m}^2$, the towel is $T\,\mathrm{m}^2$, and he starts with 1 liter of water on him, we have

$$\text{wetness after regular toweling} = \frac{1}{1+T}$$
$$\text{wetness after “split” toweling} = \frac{1}{(1+T/2)^2}.$$



   Let's see just how much this "splitting" idea will buy us.

   (a) Suppose Michael splits his towel into 3 parts, and uses all three. How wet will he be? How about if he splits it into n parts?

   (b) Use calculators to fill in the table below with 4-decimal place numbers.

T	$n=1$	$n=10$	$n=100$	$n=1000$	$n=10000$
$1\,\mathrm{m}^2$					
$2\,\mathrm{m}^2$					
$4\,\mathrm{m}^2$					
$\frac{1}{2}\,\mathrm{m}^2$					

   (c) Consider the $1\,\mathrm{m}^2$ towel. How would you describe the effect of dividing it into more and more pieces? For instance, does dividing more always make Michael dryer? Might it make him wetter? Can he get as dry as he might possibly want with that one towel? Does it even matter how big his towel is?

3. Suppose you bake a square cake, 10 inches on a side and 2 inches high. You frost it on the top and all four sides (but not the bottom). How can 6 people divide up the cake so that each gets the same amount of cake and the same amount of frosting? How about 9 people? n people?

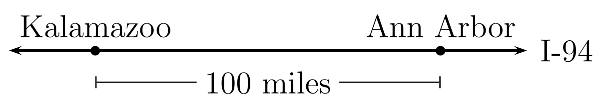
4. Last time we found that if c is the temperature we read on the Celcius thermometer, then $f = \frac{9}{5}c + 32$ is the temperature in Fahrenheit.

   Imagine yourself in a cabin in the backwoods of northern Canada. You are very cold from a long day of skiing, and you need to know the temperature in Fahrenheit, so you can tell how much antifreeze to put into your car so that it will start in the morning. Unfortunately, this is Canada, so the only thermometer available is in Celcius, and you are too cold to multiply, so you can't use the formula above. You probably couldn't even add more than two numbers together.

   In the cabin with you are a roll of duct tape, a $33\frac{1}{3}$ RPM record player, a Beatles record from 1967, and a stopwatch, among other things. How can you compute the temperature in Fahrenheit without multiplying or dividing?

5. Kalamazoo is 100 miles west of Ann Arbor along Route 94.

   Let $T(x)$ be the temperature in Fahrenheit at a point x miles west of Ann Arbor.



   (a) Define a function A in terms of T so that $A(m)$ is the temperature in Fahrenheit at a point m miles east of Kalamazoo.

   (b) Define a function B in terms of T so that $B(k)$ is the temperature in Fahrenheit at a point k **kilometers** east of Kalamazoo. (1 mile = 1.6 kilometers.)

   (c) Define a function C in terms of T so that $C(k)$ is the temperature in **Celcius** at a point k kilometers east of Kalamazoo.

6. (This problem appeared on a Fall, 2012 Math 115 exam) Suppose p represents the price of a reuben sandwich at a certain restaurant on State St. $R(p)$ represents the number of reubens the restaurant will sell in a day if they charge \$$p$ per reuben.

   (a) What does $R(5.5)$ represent in the context of this situation?

   (b) Assuming R is invertible, what does $R^{-1}(305)$ represent?

   (c) The owner of the restaurant also has a Church St. location. It doesn't get quite as much business, and the owner finds that the State St. store sells 35% more reubens than the Church St. store sells at the same price. Let $C(p)$ be the number of reubens the Church St location sells in a day at a price of \$$p$ each. Write a formula for $C(p)$ in terms of $R(p)$.

   (d) The owner starts doing research on reuben sales at the State Street location; he wants to know how the number of reubens sold is related to price. He finds that every time he raises the price by \$1 per reuben, the number sold in a day decreases by 20%. Let the constant B represent the number of reubens sold in a day at the State Street store if the price of reubens is \$5 each. Write a formula for $R(p)$ involving the constant B. Assume the domain of R is $1 \leq p \leq 25$.