Douglass Houghton Workshop, Section 1, Wed 12/06/23 Worksheet Remember What Peace There May Be in Silence

1. (Fall, 2016) Drew has a small orchard where he grows Michigan apples. After careful study last season, Drew found that the total cost, in dollars, of producing a bushels of apples can be modeled by

$$C(a) = -25500 + 26000e^{0.002a}$$

for $0 \le a \le 320$. Pearl has promised to buy up to 100 bushels of apples for her famous apple ice cream. If Drew has any remaining apples, he has an agreement to sell them to Dom's cider mill at a reduced price. Let R(a) be the revenue generated from selling a bushels of apples. Then

$$R(a) = \begin{cases} 70a & \text{if } 0 \le a \le 100\\ 2000 + 50a & \text{if } 100 < a \le 320 \end{cases}$$

- (a) How much will Dom's cider mill pay per bushel?
- (b) What is Drew's fixed cost?
- (c) For what values of a will Drew's marginal revenue equal his marginal cost?
- (d) Graph marginal revenue and marginal cost.
- (e) Assuming Drew can produce up to 320 bushels of apples, how many bushels should he produce, and what will be the maximum profit?
- 2. (This problem appeared on the Winter, 2005 Math 115 Final Exam) In a certain episode of The Simpsons, Homer needs to deliver Lisa's homework to her at school, and he must do so before Principal Skinner arrives. Suppose Homer starts from the Simpson home in his car and travels with velocity given by the figure below. Suppose that Principal Skinner passes the Simpson home on his bicycle 2 minutes after Homer has left, following him to the school. Principal Skinner is able to sail through all the traffic and travels with constant velocity 10 miles per hour.



- (a) How far does Homer travel during the 10 minutes shown in the graph?
- (b) What is the average of Homer's velocity during the 10 minute drive?
- (c) At what time, t > 0, is Homer the greatest distance ahead of Principal Skinner?
- (d) Does Principal Skinner overtake Homer, and if so, when? Explain.
- (e) If Skinner continues at 10 mph, and if Homer's velocity continues to decrease at a constant rate, then how long does it take for Homer and Skinner to meet?

3. (This problem appeared on the Fall, 2008 Math 115 Final Exam) Suppose that you are brewing coffee and that hot water is passing through a special, cone-shaped filter. Assume that the height of the conic filter is 3 in. and that the radius of the base of the cone is 2 in. If the water is flowing out of the bottom of the filter at a rate of 1.5 in³/min when the remaining water in the filter is 2 in. deep, how fast is the depth of the water changing at that instant?



4. A trough, as shown below, is to be made with a base that is 2 feet wide and 10 feet long. The sides of the trough are also 2 feet wide by 10 feet long, and are to be placed so they make an angle θ with the vertical.



- (a) What is the area, in terms of θ , of a cross section of the trough perpendicular to its long side? What is the volume of the trough?
- (b) What angle θ will give the trough the largest volume, and what is that volume? [Hint: you can always replace $\cos^2(\theta)$ with $1 - \sin^2(\theta)$.]