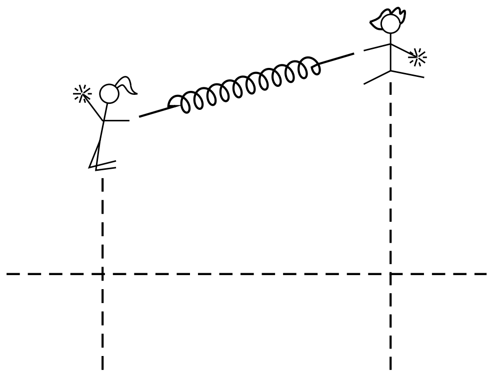
Douglass Houghton Workshop, Section 1, Mon 12/04/23

Worksheet Question Everything



1. Katie and Ashley are performing a routine in a cheerleading competition. They are both thrown straight up into the air from positions 15 feet apart, and they hold a slinky between them.

   (a) How fast is the slinky expanding when Ashley is 13.96 feet above the ground and rising at 1.6 ft/sec, while Katie is 7 feet above the ground and falling at 14 ft/sec?

   (b) Katie is thrown upward with an initial velocity of 18 ft/sec at time 0, and Ashley is thrown .3 seconds later with initial velocity 24 ft/sec. Both start 5 feet above the ground and are subject to the acceleration of gravity, which is $-32\,\text{ft/sec}^2$. Find formulas for their heights at time t.

   (c) Find the maximum and minimum length of the slinky between the time Ashley lifts off to the time Katie is caught by her teammates (at the same height they threw her from).

2. Jon is studying a colony of *campylobacter jejuni* bacteria. He finds that the growth rate of the colony is increasing exponentially. That is, if $P(t)$ is the population in thousands after t hours, then $P'(t) = Ae^{kt}$ for some constants A and k.

   (a) Suppose there are 1000 bacteria at the start of the experiment. Write an integral which gives the number of bacteria present after T hours.

   (b) Use the Fundamental Theorem of Calculus to get a formula without an integral for the number of bacteria after T hours.

   (c) Suppose the bacteria grew at an initial rate of 500 bacteria per hour, and after 6 hours the rate has increased to 1000 bacteria per hour. Find values for the constants A and k.

   (d) How many bacteria are there 6 hours after the experiment started?

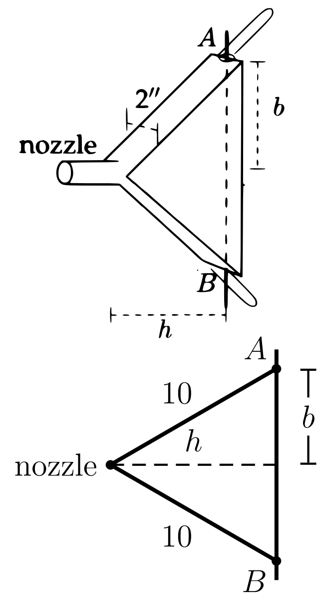
3. Kae also does an experiment with the same starting population of bacteria, but she plays rap music to the bacteria. She finds the bacteria LOVE rap music, and they grow while a song is playing and stop growing between songs. Kae plays them a series of 3-minute songs with 3-minute breaks between them, and finds that t hours after the experiment starts, the growth rate (in thousands per hour) is $1 + \sin(20\pi t)$. How many bacteria grow in each 6-minute cycle?

4. (This problem appeared on a Winter, 2016 Math 115 Final Exam.) Consider the family of functions given by $f(x) = e^{x^2+Ax+B}$ for constants A and B.

   (a) Find and classify all local extrema of $f(x) = e^{x^2+Ax+B}$. Your answers may depend on A and/or B.

   (b) Find the values of A and B that make $(3, 1)$ a critical point of $f(x)$.

5. (This problem appeared on a Winter, 2008 Math 115 exam) A bellows has a triangluar frame made of three rigid pieces. Two pieces, each 10 inches long, are hinged at the nozzle. They are attached to the third piece at points A and B which can slide, as shown in the diagrams below. (The figures show a 3D sketch of the bellows and a 2D sketch that may be specifically useful to solve the problem.)
   Each piece of the frame is 2 inches wide, so the volume (in cubic inches) of air inside the bellows is equal to the area (in square inches) of the triangluar cross-section above, times 2. Suppose you pump the bellows by moving A downward toward the center at a constant speed of 3 in/sec. (So B moves upwards at the same speed.) What is the rate at which air is being pumped out when A and B are 12 inches apart? (So A is 6 inches from the center of the vertical piece of the frame.)



6. (From the Winter, 2007 Math 115 Final Exam) Suppose that f and g are continuous functions with
$$\int_0^2 f(x)\,dx = 5 \qquad \text{and} \qquad \int_0^2 g(x)\,dx = 13.$$
   Compute the following. If the computation cannot be made because something is missing, explain clearly what is missing.

   (a) $\int_4^6 f(x-4)\,dx$

   (b) $\int_{-2}^0 2g(-t)\,dt$

   (c) $\int_2^0 (f(y)+2)\,dy$

   (d) $\int_2^2 g(x)\,dx$

   (e) Suppose that f is an even function. Find the average value of f from -2 to 2.