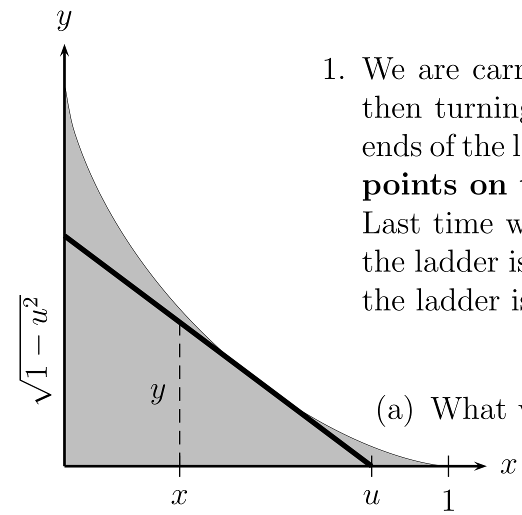
Douglass Houghton Workshop, Section 1, Wed 11/29/23

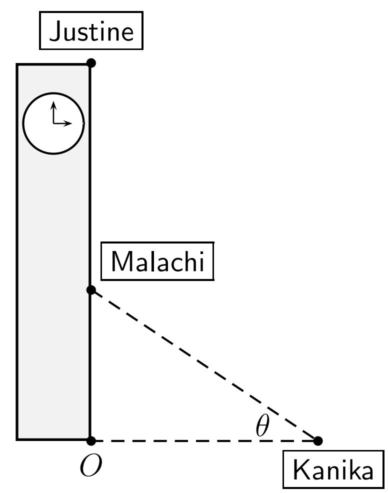
Worksheet Panda Bear



1. We are carrying a ladder of length 1 down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?**
   Last time we found that if $0 \le x \le u \le 1$ and the base of the ladder is at $(u, 0)$, then the distance from $(x, 0)$ north to the ladder is
   $$y = \frac{u-x}{u}\sqrt{1-u^2}.$$
   (a) What value of u maximizes y? (Keep x fixed!)

   (b) So for a fixed x, what is the maximum value of y, as the ladder moves?

   (c) You have found a formula for the curve at the top of the region we want. Simplify until it's beautiful. (This is the best part, so don't stop until it's truly wondrous.)



2. (Adapted from a Winter, 2005 Math 115 exam) One day Justine notices that the door to the Burton Tower carillon has been left open. She can't resist the urge to climb to the top of the tower and barricade herself in, then begin playing all her favorite Taylor Swift albums across campus.
   The University charges Malachi, as part of his ROTC training, with taking back the tower, so he begins to climb it. Meanwhile on the ground, 30 feet from the tower, Kanika stands back and plays along with Taylor on the guitar, but occasionally slips in some Pink Floyd and Led Zepplin licks.. She looks up at an angle θ to see Malachi.

   (a) Find the rate of change of Malachi's distance from the point O with respect to θ.

   (b) If the distance from point O to Justine is 200 ft and Malachi climbs at a constant 8 ft/sec, what is the rate of change of θ with respect to time when Malachi is halfway up?

   (c) When Malachi is halfway up, Justine drops the end of a a rope down to help him. The end of the rope falls with a constant acceleration of 32 ft/sec^2. When does Malachi catch it, and what is its speed when he does?

   (d) Kanika watches the end of the rope as it drops, and also begins backing away from the tower at a rate of 5 ft/sec. How fast is the angle of her gaze changing when Malachi catches the rope?

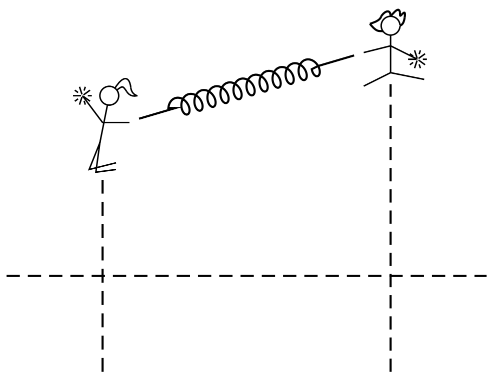
3. (Winter, 2010) Suppose that the standard price of a round-trip plane ticket from Detroit to Paris, purchased t days after April 30, is $P(t)$ dollars. Assume that P is an invertible function (even though this is not always the case in real life). In the context of this problem, give a practical interpretation for each of the following:

   (a) $P'(2) = 55$

   (b) $\int_5^{10} P'(t)\, dt$

   (c) $P^{-1}(690)$

   (d) $\frac{1}{5}\int_5^{10} P(t)\, dt$



4. Katie and Ashley are performing a routine in a cheerleading competition. They are both thrown straight up into the air from positions 15 feet apart, and they hold a slinky between them.

   (a) How fast is the slinky expanding when Ashley is 13.96 feet above the ground and rising at 1.6 ft/sec, while Katie is 7 feet above the ground and falling at 14 ft/sec?

   (b) Katie is thrown upward with an initial velocity of 18 ft/sec at time 0, and Ashley is thrown .3 seconds later with initial velocity 24 ft/sec. Both start 5 feet above the ground and are subject to the acceleration of gravity, which is $-32\,\text{ft/sec}^2$. Find formulas for their heights at time t.

   (c) Find the maximum and minimum length of the slinky between the time Ashley lifts off to the time Katie is caught by her teammates (at the same height they threw her from).

5. (Fall 2008) This problem was a smörgåsbord:

   (a) If $f(x)$ is even and $\int_{-2}^{2}(f(-x) - 3)\, dx = 8$, find $\int_0^2 f(x)\, dx$.

   (b) The average value of the function $g(x) = 10/x^2$ on the interval $[c, 2]$ is equal to 5. Find the value of c.

   (c) If people are buying UMAir Flight 123 tickets at a rate of $R(t)$ tickets/hour (where t is measured in hours since noon on December 15, 2008), explain in words what $\int_3^{27} R(t)\, dt$ means in this context.

   (d) Suppose that the function $N = f(t)$ represents the total number of students who have turned in this exam t minutes after the beginning of the exam. Interpret $(f^{-1})'(325) = 2$.

   (e) Find k so that the function $h(x)$ below is continuous for all x.

$$h(x) = \begin{cases} x^2 - 1 & \text{if } x \le 1 \\ 6\sin(\pi(x - 0.5)) + k & \text{if } x > 1 \end{cases}$$