**Douglass Houghton Workshop, Section 1, Mon 11/27/23**

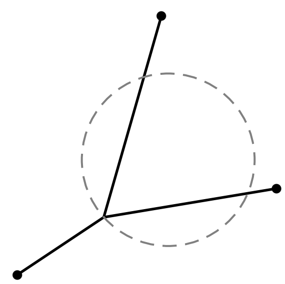
# Worksheet Once More, Unto the Breach, Dear Friends

1. Shortest Network. Last time we used calculus to show that a $\Lambda$-shaped network can be improved if the vertex angle is less than $120°$.

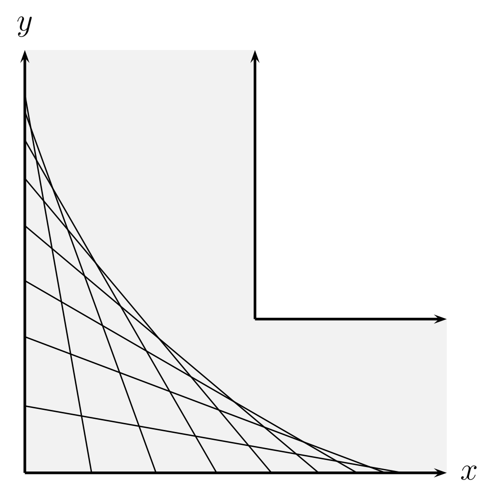
   (a) Prove that the network to the right is NOT minimizing. You don't need to find the optimal network, just prove that this one can be improved. Hint: consider the portion of the network that is inside the circle.

   (b) What allowed that trick to work? Phrase your answer like this: "Any network which contains ______ can be improved."

   (c) Put it all together, and explain where the soap puts the roundabout.



2. Suppose we are carrying a ladder down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?** Let's assume the ladder has length 1. In the picture, the gray area is the hallway and the fine lines are the ladder in different positions.

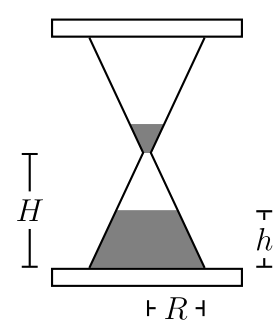



   (a) Suppose the base of the ladder is at the point $(u, 0)$. Where on the $y$-axis is the top of the ladder? Draw a picture!

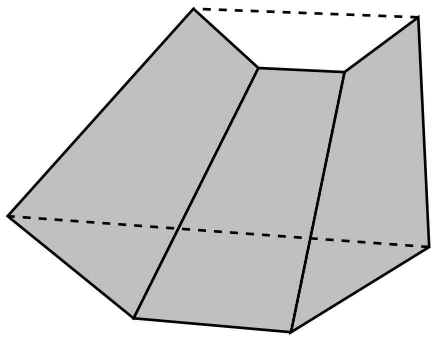
   (b) Suppose you are standing at $(x, 0)$ and looking north (up the page). If $x < u$, how far away do you see the ladder?

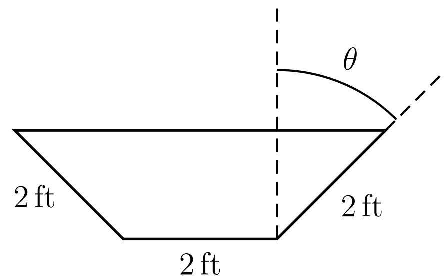
   To be continued...

3. The lower chamber of an hourglass is shaped like a cone with height $H$ inches and base radius $R$ inches, as shown in the figure to the right, above. Sand falls into this cone. Write an expression for the volume of the sand in the lower chamber when the height of the sand there is $h$ in (Hint: A cone with base radius $r$ and height $y$ has volume $V = \frac{1}{3}\pi r^2 y$, and it may be helpful to think of a difference between two conical vol umes.) Then, if $R = 0.9$ in, $H = 2.7$ in, and sand is falling into the lower chamber at $2\,\text{in}^3/\text{min}$, how fast is the height of the sand in the lower chamber changing when $h = 1$ in?



4. A trough, as shown below, is to be made with a base that is 2 feet wide and 10 feet long. The sides of the trough are also 2 feet wide by 10 feet long, and are to be placed so they make an angle $\theta$ with the vertical.



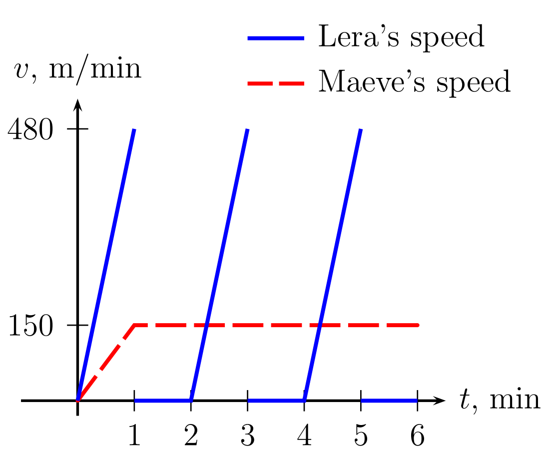



(a) What is the area, in terms of $\theta$, of a cross section of the trough perpendicular to its long side? What is the volume of the trough?

(b) What angle $\theta$ will give the trough the largest volume, and what is that volume? [Hint: you can always replace $\cos^2(\theta)$ with $1 - \sin^2(\theta)$.]

5. (Adapted from a Winter 2009 Math 115 Exam.) Maeve and Lera, after much friendly trash talk about who's the fastest runner, decide to have a race. The two employ very different approaches.

- Maeve takes the first minute to accelerate to a slow and steady pace which she maintains through the remainder of the race.
- Lera, on the other hand, spends the first minute accelerating to faster and faster speeds until she's exhausted and has to stop and rest for a minute—and then she repeats this process until the race is over. The graph below shows their speeds (in meters per minute), $t$ minutes into the race. (Assume that the pattern shown continues for the duration of the race.)




(a) What is Maeve's average speed over the first two minutes of the race? What is Lera's?

(b) Lera immediately gets ahead of Maeve at the start of the race. How many minutes into the race does Maeve catch up to Lera for the first time?

(c) Draw graphs of Lera's and Maeve's positions at time $t$. Be as precise as possible.

(d) If the race is 720 meters total, who wins? What if it's 721 meters?