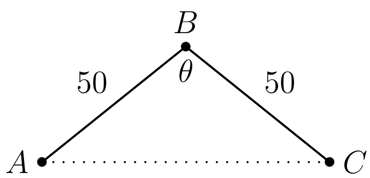
Douglass Houghton Workshop, Section 1, Mon 11/20/23

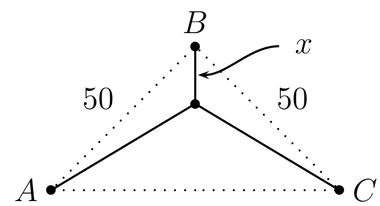
Worksheet Now is the Winter of our Discontent

1. Shortest Network. So far we've looked at the case where the cities are at the corners of an isosceles triangle like the one below. We know:



- When angle B is $70°$ or $90°$, it is possible to improve upon the Λ-shaped network shown by building a roundabout south of B and connecting it to all three cities.
- However, when B is $150°$, the Λ is better than all possible ⅄'s.

   (a) Suppose the measure of angle B is θ. Use the law of cosines to write a formula for the length of the ⅄-shaped network to the right, in terms of θ and x.



   (b) Call that function $L_\theta(x)$. Put your calculator in degrees mode and plot $L_{70}(x)$, $L_{90}(x)$, and $L_{150}(x)$, for x from 0 to 50. Put the graphs on the board.

   (c) The shapes seem about the same. What can you see in the graphs that explains the fact that in two cases the Λ can be improved, and in the other it can't? (Remember the Λ is $x = 0$.)

   (d) Use calculus to figure out which Λ's can be improved, and which can't. State the result in the form: "Any Λ-shaped network with an angle smaller than ______ can be improved".

   (e) This is for those who like to compute and simplify. Show that the function $L_\theta(x)$ defined above is always concave up, by finding and simplifying its second derivative.

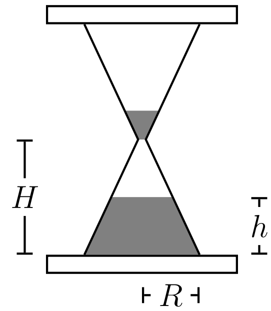
2. (Adapted from a Fall, 2004 Math 115 final) Jonathan spends Thanksgiving eating poutine. The rate at which he eats poutine is given by the function $r(t)$, where t is measured in hours and $r(t)$ is in liters of poutine/hour. Suppose $t = 0$ corresponds to 10 am, when the binge begins.

   (a) Write a definite integral that represents the total amount of poutine Jonathan consumes between noon and 10 pm.

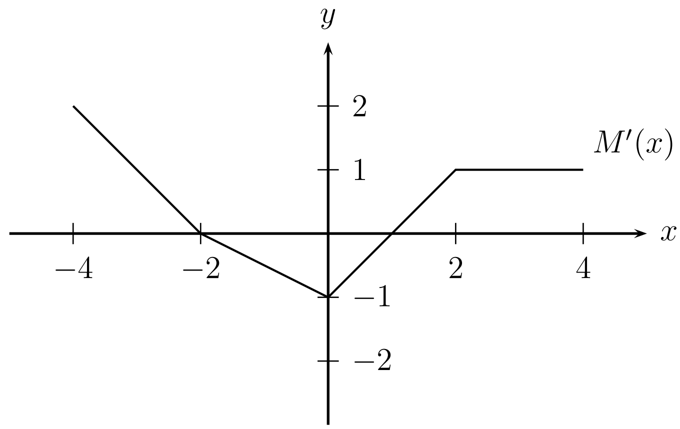
   (b) If Jonathan's rate of eating poutine is given by $r(t) = e^{-t} + 1$, use a left hand sum with three (3) subdivisions to estimate the amount of poutine Jonathan eats in the first four hours of his binge.

   (c) Should your estimate in part (b) be an underestimate or an overestimate? Explain.

3. The lower chamber of an hourglass is shaped like a cone with height H inches and base radius R inches, as shown in the figure to the right, above. Sand falls into this cone. Write an expression for the volume of the sand in the lower chamber when the height of the sand there is h in (Hint: A cone with base radius r and height y has volume $V = \frac{1}{3}\pi r^2 y$, and it may be helpful to think of a difference between two conical vol umes.) Then, if $R = 0.9$ in, $H = 2.7$ in, and sand is falling into the lower chamber at $2\,\text{in}^3/\text{min}$, how fast is the height of the sand in the lower chamber changing when $h = 1$ in?



4. Here is the graph of the *derivative* of the continuous function $M(x)$. Using the fact that $M(-4) = -2$, sketch the graph of $M(x)$. Give the coordinates of all critical points, inflection points, and endpoints.



5. Suppose $\int_4^9 (4f(x) + 7)\,dx = 315$. Find $\int_4^9 f(x)\,dx$.

6. (From the Winter, 2007 Math 115 Final Exam) Suppose that f and g are continuous functions with

$$\int_0^2 f(x)\,dx = 5 \qquad \text{and} \qquad \int_0^2 g(x)\,dx = 13.$$

Compute the following. If the computation cannot be made because something is missing, explain clearly what is missing.

(a) $\int_4^6 f(x-4)\,dx$

(b) $\int_{-2}^0 2g(-t)\,dt$

(c) $\int_2^0 (f(y) + 2)\,dy$

(d) $\int_2^2 g(x)\,dx$

(e) Suppose that f is an even function. Find the average value of f from -2 to 2.