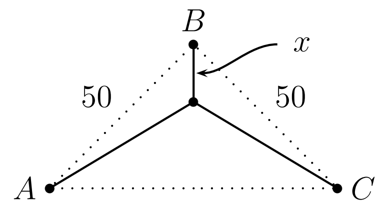
Douglass Houghton Workshop, Section 1, Wed 11/15/23

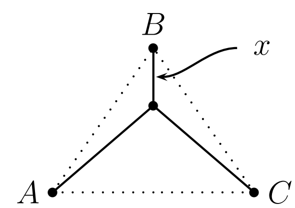
Worksheet May the Road Rise to Meet You

1. We've been working on the problem of finding the shortest road network between three cities in the plane.

   In the case we considered, the three cities were at the corners of a 45°-45°-90° triangle with legs 50 miles long. The simplest idea is to just build roads along the legs; that makes a network of length 100. But by constructing a ⅄-shaped network like the one at the right, we found

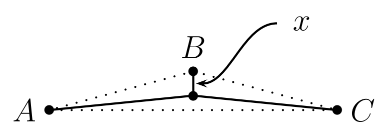
   

   - The length of the network is $x + 2\sqrt{2500 - 100x\cos(45) + x^2}$.
   - We can improve from the simple 2-road solution ($x = 0$, length $= 100$) by increasing x. For instance, when $x = 10$, the network has a length of about 97.

   (a) Consider the case where the triangle is still isosceles and the legs still have length 50, but the angle at B is 70°. Write a formula for the length of the network.

   (b) Can you find a value of x which beats the 2-road solution ($x = 0$, length $= 100$)?

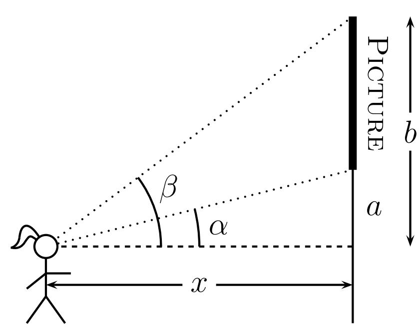
   

   (c) Now suppose the vertex angle is very obtuse—say 150°. Find a formula for the length of the network.

   (d) Can you beat the 2-road solution in this case?

   (e) Suppose the vertex angle is θ. Write a formula for the length of the network.

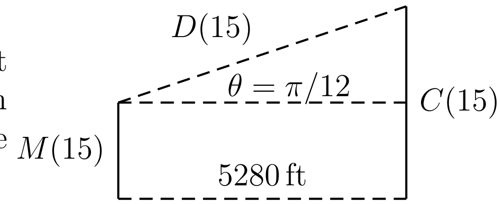
2. Yeji has drawn a beautiful cartoon of Ava with her 5 cats, Calliope, Cassiopea, Cantelope, Cleopatra, and Shadow. The cartoon has been blown up and framed, and now hangs high on the wall of a gallery in Ann Arbor. Its bottom is a feet above eye level, and its top is b feet above eye level. If you stand far away from the wall, you can't see the picture well. But if you stand close to the wall, you can't see well either! So the question is: how far from the wall should you stand in order to have the best view?

   (a) Let α and β be the angles between eye level and the bottom and top of the picture, as shown. x is your distance from the wall. Find α and β in terms of x, a, and b.

   (b) $\beta - \alpha$ is the angle that the picture takes up in your field of vision. So find the value of x that maximizes $\beta - \alpha$.

3. (Adapted from a Fall, 2011 Math 115 Final Exam problem) Maeve takes the train back to Ann Arbor from Washington after Thanksgiving. At one point during the trip the tracks run parallel to a road, which is a mile away. The train is going quite slowly (6 ft/sec). Maeve spots a Maserati sports car even with the train on the road, and turns her head as she watches it pull ahead. Let $C(t)$ be the distance between the car and its starting point, and $M(t)$ be Maeve's distance from her starting point. After watching the car for 15 seconds, Maeve has rotated her head $\pi/12$ radians.



   (a) Initially the car is 1 mile (5280 ft) due east of the train. Find the distance between Maeve and the car 15 seconds after she starts watching it.

   (b) Let $\theta(t)$ be the angle Maeve has turned her head after tracking the car for t seconds. Write an equation for the distance between Maeve and the car at time t. (Your answer may involve $\theta(t)$.)

   (c) If at precisely 15 seconds, Maeve is turning her head at a rate of .01 radians per second, what is the instantaneous rate of change of the distance between Maeve and the car?

   (d) What is the speed of the car at 15 seconds?

4. The table below gives the expected growth rate, $g(t)$, in ounces per week, of the weight of a baby in its first 54 weeks of life (which is slightly more than a year). Assume for this problem that $g(t)$ is a decreasing function.

week t	0	9	18	27	36	45	54
growth rate $g(t)$	6	6	4.5	3	3	3	2

   (a) Using six subdivisions, find an overestimate and underestimate for the total weight gained by a baby over its first 54 weeks of life.

   (b) How often would you have to weigh the baby to get an estimate guaranteed to be accurate to within $\frac{1}{4}$ pound?

5. Here is the graph of the *derivative* of the continuous function $M(x)$. Using the fact that $M(-4) = -2$, sketch the graph of $M(x)$. Give the coordinates of all critical points, inflection points, and endpoints.