Douglass Houghton Workshop, Section 1, Mon 10/23/23

Worksheet Kangaroo

1. In "The 12 days of Christmas", a certain poultry-afficianado receives a number of gifts from her true love:

   Day 1: A partridge in a pear tree. How to get it down?

   Day 2: 2 turtle doves, and another partridge in a pear tree. Is it the same tree?

   Day 3: 3 French hens, 2 more turtle doves, and another partidge.

   ...

   Day 12: 12 drummers drumming (loudly), eleven pipers piping (make them stop!), ..., and yet another partridge in a pear tree.

   (a) How many total partridges does the heroine receive, over the course of the song? How many turtle doves?

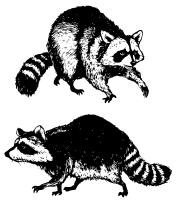
   (b) If item 1 is "partridge", item 2 is "turtle dove", etc., then write a formula for the total number of item n's received.

   (c) Of which item does Mr. Truelove send the most? (Solve using calculus.)

2. Recall that Jess is studying a population of raccoons in New York City. Last time the population changed according to the rule:

   $$P(n+1) = 1.5P(n) - 200$$

   where $P(0)$ is the population in 2023, $P(1)$ is the population 1 year later, etc. (P is measured in raccoons.)

   We found a nice way to visually understand how the population changes over time:

   (a) Draw a big set of axes, where x and y go from -200 to 600, and on them, in two different colors, show the lines $\boxed{y = x}$ and $\boxed{y = 1.5x - 200}$.

   (b) For a population starting at 320 raccoons, find $(320, 320)$ on the graph of $y = x$.

   (c) Go up or down to the other line, then left or right back to $y = x$. That's the next year's population.

   (d) Repeat several times to see what happens in the long run.

   (e) Now start the population at 440, and repeat the last two steps so show what happens in the long run.

   (f) Give a concise statement about the fate of the raccoons.

3. Repeat the last problem, but for the rule

   $$P(n+1) = .75P(n) + 125.$$

4. A population equilibrium is **stable** if the population moves toward the equilibrium, rather than away from it. Which of the last two scenarios has a stable equilibrium?

5. (This problem appeared on the Fall, 2008 Math 115 Final Exam) At the Michigan-Ohio State basketball game this year, the Michigan Band discovers that the amount of time it spends playing "Hail to the Victors" has a direct impact on the number of points our team scores. If the band plays for x minutes, then the Wolverines will score

$$W(x) = -.48x^2 + 7.2x + 63$$

points. Assume that the band can play for a maximum of 10 minutes.

   (a) How long should the band play to maximize the number of points Michigan scores?

   (b) The band affects how many points Ohio State scores as well. x minutes of playing results in the Buckeyes scoring

$$B(x) = -x^2 + 8x + 84$$

   points. Find the number of minutes the band should play to maximize the margin of victory for Michigan.

   (c) What will be the score of the game for the case you found in part (b)?

6. Faith and Mateo drive on the New York Thruway for a track meet. They enter at Buffalo, where a camera takes a picture of the car's license plate. The picture is processed and the licensce plate stored in a database, along with the time the picture was taken. Later Faith and Mateo exit at Albany, where another picture is taken. A week afterward, they receive a bill in the mail along with a speeding citation, stating that they were going exactly 75 mph at some point on their trip. How does the Mean Value Theorem allow the authorities to be sure that happened?

7. (This problem appeared on a Winter, 2008 Math 115 Exam.)

   (a) Consider the function $f(x) = x\sqrt{x+1}$. What is the domain of f?

   (b) Find all critical points, local maxima, and local minima of f.

   (c) Which of the local maxima and minima are global maxima / minima?

8. The diagrams below each have 4 regions, representing different ways a function can behave at a point. In each region write an example of a function and a point that meets the criteria. For example, in the intersection of "$f' = 0$" and "f changes direction", we have $x^2 \,@\, x = 0$, because the derivative of x^2 is indeed 0 at $x = 0$, and the function switches from decreasing to increasing there.