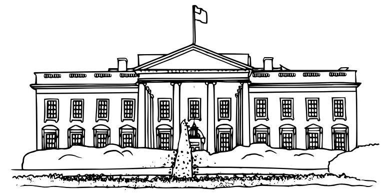
Douglass Houghton Workshop, Section 1, Wed 10/18/23

Worksheet Joy is not in things; it is in us



1. We still have this $1/z$ scale model of the White House, which we plan on blowing up. We want to decide what speed to run the film at, so that when we slow it down to 24 frames per second, we get a realistic explosion.

   (a) Last time we showed that an object will fall $16t^2$ feet in t seconds. So how long does it take for an object to fall off the real white house, which is H feet tall? How many frames will that be, if we film it at 24 frames per second and show it at the same speed?

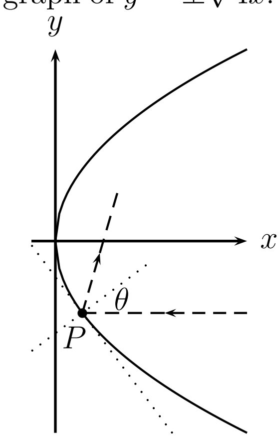
   (b) How long does it take an object to fall off the top of the model?

   (c) How many frames per second should you film to get the right number of frames to make it look like the model is full-sized?

2. Last time we thought about a parabolic mirror in the shape of the graph of $y = \pm\sqrt{4x}$.

   So far we've found:

   - A light ray $y = -b$ hits the mirror at $P = (b^2/4, -b)$.
   - The slope of the tangent at that point is $-2/b$.
   - The normal line at the same point has slope $b/2$.
   - When a line makes an angle θ with the x-axis, it has slope $\tan\theta$.
   - So if we call the angle between the normal line and the horizontal θ, then $\theta = \tan^{-1}(b/2)$.



   (a) Draw the picture on the board.

   (b) To the ray, the mirror looks flat, just like the tangent line. Draw the reflected ray. What angle does it make with the x-axis?

   (c) We know that $\sin 2x = 2\sin x \cos x$ and $\cos 2x = \cos^2 x - \sin^2 x$. Use those to find a formula for $\tan 2x$ in terms of $\tan x$.

   (d) What is the slope of the reflected ray?

   (e) Write an equation for the reflected ray.

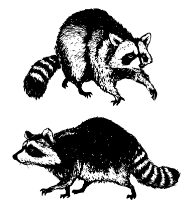
   (f) Where does the reflected ray intersect the x-axis? What is surprising about this answer?

   (g) Graph several rays, with their reflections.

   (h) What's cool about this type of mirror?

3. Let $f^{(n)}(x)$ denote the nth derivative of f. If $f(x) = e^{-2x}$, find $f^{(531)}(x)$. Is $f^{(531)}(x)$ increasing or decreasing? Concave up or concave down? Try graphing $f^{(531)}$ without your calculator, then check with the calculator.

4. Jess is studying a population of raccoons in New York City. Suppose that the population changes according to the rule:



$$P(n+1) = 1.5P(n) - 200$$

where $P(0)$ is the population in 2023, $P(1)$ is the population 1 year later, etc. (P is measured in raccoons.)

   (a) Make up a (short) story about raccoons that yields that formula as the result.

   (b) Suppose $P = 320$ in 2023. What will happen in the long run?

   (c) Suppose instead that $P = 440$ in 2023. Now what happens?

   (d) A population is in **equilibrium** if it stays the same from year to year. Is there an equilibrium number for this population?

   (e) Explain these results pictorially by drawing the graphs of $y = x$ and $y = 1.5x - 200$. Start at $(320, 320)$, go down to the other graph, and then over to $y = x$. That's the new population. Repeat. Then start at 440.

5. Repeat the last problem, but for the rule

$$P(n+1) = .75P(n) + 125.$$

6. (An old team homework problem.) Let $f(x) = x^2 - 2x + 13$ and $g(x) = -x^2 - 2x - 5$.

   (a) Draw $y = f(x)$ and $y = g(x)$ on the same set of axes. How many lines are tangent to both graphs?

   (b) Find the equations of those lines.

7. (This problem appeared on a Winter, 2004 Math 115 exam. Really!) While exploring an exotic spring break location, you discover a colony of geese who lay golden eggs. You bring 20 geese back with you. Suppose each goose can lay 294 golden eggs per year. You decide maybe 20 geese isn't enough, so you consider getting some more of these magical creatures. However, for each extra goose you bring home there are less resources for all the geese. Therefore, for each new goose the amount of eggs produced will decrease by 7 eggs per goose per year. How many more geese should you bring back if you want to maximize the number of golden eggs per year laid?

8. (This problem appeared on a Fall, 2006 Math 115 exam) The Flux F, in millilitres per second, measures how fast blood flows along a blood vessel. Poiseuille's Law states that the flux is proportional to the fourth power of the radius, R, of the blood vessel, measured in millimeters. In other words $F = kR^4$ for some positive constant k.

   (a) Find a linear approximation for F as a function of R near $R = 0.5$. (Leave your answer in terms of k).

   (b) A partially clogged artery can be expanded by an operation called an angioplasty, which widens the artery to increase the flow of blood. If the initial radius of the artery was 0.5 mm, use your approximation from part (a) to approximate the flux when the radius is increased by 0.1 mm.

   (c) Is the answer you found in part (b) an under- or over-approximation? Justify your answer.