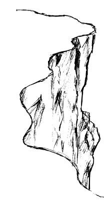
Douglass Houghton Workshop, Section 1, Wed 10/11/23

Worksheet If Wishes Were Changes, We'd All Live in Roses

1. One beautiful day, Drew and his twin brother Adam are playing basketball at the top of a cliff, when the ball accidentally rolls to the edge of the cliff and falls over.

   (a) The basketball's downward acceleration due to gravity is $-32\,\text{ft}/\text{sec}^2$. So what is its velocity t seconds after it is dropped? (Keep in mind that the derivative of velocity is acceleration, and the initial velocity is 0.)

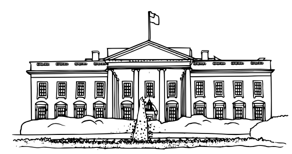
   

   (b) How far has the basketball fallen after t seconds?

   (c) Suppose the basketball takes 3.5 seconds to hit the bottom. How high is the cliff?

   (d) Explain a general method for finding the height of any cliff.

2. Suppose you construct a $1/z$ scale model of the White House, in order to film it blowing up. You will show the film at 24 frames per second. How many frames per second should you *film* so that when you slow the speed down, things will fall at believable speeds?

3. Consider a mirror in the shape of the graph of $y = \pm\sqrt{4x}$.

   (a) Draw the mirror (make it big). What shape is it?

   (b) Draw a light ray travelling leftward along the line $y = -b$, where b is some positive number (making $-b$ negative). At what point P does the ray hit the mirror?

   (c) Find, in terms of b, the slope of the tangent to the mirror at P.

   (d) The *normal* to a curve at a point is the line through that point which is perpendicular to the tangent line. Find the slope of the normal to the mirror at P, and draw both the normal and tangent lines on your graph.

   (e) Suppose a line makes an angle θ with the positive x-axis. What is the slope of the line?

   (f) Let θ be the angle the normal to the mirror at P makes with the light ray $y = -b$. Can you write θ in terms of b? Hint: Use (3d) and (3e).

   To be continued...

4. (From Fall, 2011, Math 115 Exam 2) Let $f(x) = \ln(x)$. Use the table of values below for $g(x)$ and $g'(x)$ to answer the following questions.

   (a) If $F(x) = f(g(x))$, find $F'(4)$.

   (b) If $G(x) = g^{-1}(x)$, find $G'(4)$.

   (c) If $H(x) = \tan(g(x))$, find $H'(3)$.

   (d) If $E(x) = e^{f(x)g(x)}$, find $E'(2)$.

x	2	3	4
$g(x)$	1	4	6
$g'(x)$	5	3	2

5. (An old team homework problem.) Let $f(x) = x^2 - 2x + 13$ and $g(x) = -x^2 - 2x - 5$.

   (a) Draw $y = f(x)$ and $y = g(x)$ on the same set of axes. How many lines are tangent to both graphs?

   (b) Find the equations of those lines.

6. (This problem appeared on a Winter 2007 Math 115 exam) Suppose f and g are differentiable functions with values given by the table below.

   (a) If $h(x) = f(x)g(x)$, find $h'(3)$.

   (b) If $j(x) = \dfrac{(g(x))^3}{f(x)}$, find $j'(1)$.

   (c) If $d(x) = x \ln\left(e^{f(x)}\right)$, find $d'(3)$.

   (d) If $t(x) = \cos(g(x))$, find $t'(1)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	9	-3	7
3	4	11	15	-19

7. (This is based on an old web homework problem) Consider the two functions $f(x) = \sin(x)$ and $g(x) = e^{k-x}$. We want to find the smallest positive k that makes the two graphs tangent.

   (a) Draw both functions on the same set of axes, using $k = 1$.

   (b) Imagine you have a dial to turn, and the dial controls the value of k. When you increase k, how does the picture change? What we want to do is turn the dial to just the right place. Imagine what that would look like.

   (c) Let a be the x-value of the point of tangency. Write down two equations for what must be true at a if k is just right.

   (d) Solve for k and a.

   (e) Graph your two functions to see if they really are tangent.