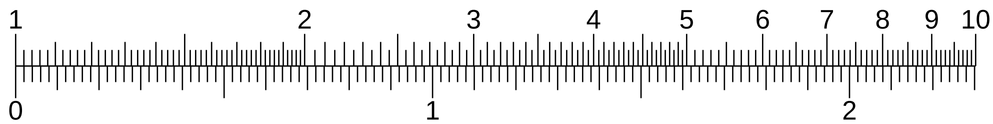
Douglass Houghton Workshop, Section 1, Wed 09/20/23

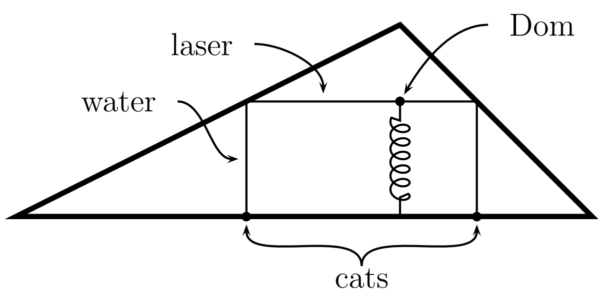
Worksheet Go Forth and Multiply

1. Explain how to use two rulers to add numbers.

2. Explain how a slide rule is able to multiply two numbers. This picture may be helpful:



3. Dom is being raised on a giant spring-loaded platform right under the peak of a triangular room. Laser beams are being emitted from his head, parallel to the ground, until they hit the walls. Where they hit the walls, drops of water fall down, then land in the mouths of two cats. As Dom goes up, the cats follow the drops toward the base of the platform.



   (a) Let $h(t)$ be Dom's height at time t, and let $w(t)$ be the distance between the two cats. Are they continuous functions? Is $h(t) - w(t)$ a continuous function?

   (b) When t is close to 0 (so Dom's head has just come through the floor), what can you say about $h(t) - w(t)$?

   (c) Later on, when Dom is near the end of his journey and about to hit the top, what can you say about $h(t) - w(t)$?

   (d) Use the Intermediate Value Theorem to show that at some time the distance between the cats is the same as Dom's height off the floor.

4. Prove that it's possible to make a fair five-sided die. Rules:

   (a) All sides must be flat,

   (b) It must be equally likely to land on all sides, and

   (c) No handles (ala a dreidel).

5. Why is it necessary to define the derivative in terms of a limit? Draw a picture that describes how the derivative is the limit of the slopes of some lines.

6. The *power rule for derivatives* says that if $f(x) = x^n$, then $f'(x) = nx^{n-1}$. Use the definition of the derivative to prove it for the case where n is a positive integer. Hint: Pascal's triangle.

7. (This problem appeared on a Fall, 2010 Math 115 exam.) Before the industrial era, the carbon dioxide (CO_2) level in the air in Ann Arbor was relatively stable with small seasonal fluctuations caused by plants absorbing CO_2 and producing oxygen in its place. Typically, on March 1, the CO_2 concentration reached a high of 270 parts per million (ppm), and on September 1, the concentration was at a low of 262 ppm. Let $G(t)$ be the CO_2 level t months after January 1.

   (a) Assuming that $G(t)$ is periodic and sinusoidal, sketch a neat, well-labeled graph of G with $t = 0$ corresponding to January 1.

   (b) Determine an explicit expression for G, corresponding to your sinusoidal graph above.

8. (This problem appeared on a Winter, 2014 Math 115 Exam.) The air in a factory is being filtered so that the quantity of a pollutant, P (in mg/liter), is decreasing exponentially. Suppose t is the time in hours since the factory began filtering the air. Also assume 20% of the pollutant is removed in the first five hours.

   (a) What percentage of the pollutant is left after 10 hours?

   (b) How long is it before the pollution is reduced by 50%?

9. (This problem appeared on a Winter, 2016 Math 115 exam) Consider the function $f(x)$ defined by

$$f(x) = \begin{cases} xe^{Ax} + B & \text{if } x < 3 \\ C(x-3)^2 & \text{if } 3 \le x \le 5 \\ \frac{130}{x} & \text{if } x > 5 \end{cases}$$

   Suppose that $f(x)$ is continuous at $x = 3$, $\lim_{x\to 5^+} f(x) = 2 + \lim_{x\to 5^-} f(x)$, and $\lim_{x\to -\infty} f(x) = -4$. Find A, B, and C.

10. (This problem appeared on a Winter, 2012 Math 115 exam. Really!) Enjoying breakfast outdoors in a coastal Mediterranean town, Tommy notices a ship that is anchored offshore. The ship is stationed above a reef which lies below the surface of the water, and a series of waves causes its height to oscillate sinusoidally with a period of 6 seconds. When Tommy begins observing, the hull of the ship is at its highest point, 20 feet above the reef. After 1.5 seconds, the hull is 11 feet above the reef.

   (a) Write a function $h(t)$ that gives the height of the ship's hull above the reef t seconds after Tommy begins observing.

   (b) According to your function, will the hull of the ship hit the reef? Explain.

   (c) What proportion of the time is the ship within 4 feet of the reef?