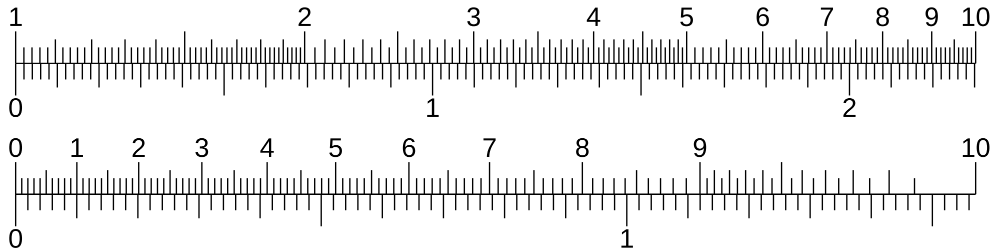
Douglass Houghton Workshop, Section 1, Mon 09/18/23

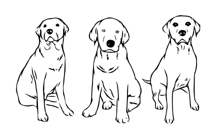
Worksheet Friends, Romans, and Countrymen

1. What's the deal with these pictures? What are they good for?



2. Nicholas has noticed that his tastes changed over the last year. A year ago he spent about 15 hours a week preparing for the Math Olympiad, and 10 hours playing with his dogs Ace, Kimmy, Benz, Benz Jr., Cookie, and Brownie. Gradually school took over his life, and though there have been some ups and downs in his schedule, the general trend is that he's spent less time per week on both. Now, 52 weeks later, he spends only 3 hours a week on the Math Olympiad and 5 hours a week playing with his dogs.

   Let $M(t)$ be the number of hours Nicholas spent on the Math Olympiad in week t, and let $D(t)$ be the number of hours he spent playing with his dogs. Assume $M(t)$ and $D(t)$ are continuous functions of time.

   (a) What does it mean for a function to be continuous?

   (b) Are $M(t) + D(t)$, $M(t) - D(t)$, and $M(t)D(t)$ continuous?

   (c) Use one of the functions in part (a) together with the Intermediate Value Theorem to show that at some time in the last year, Nicholas was spending the same amount of time on the Math Olympiad and playing with his dogs.

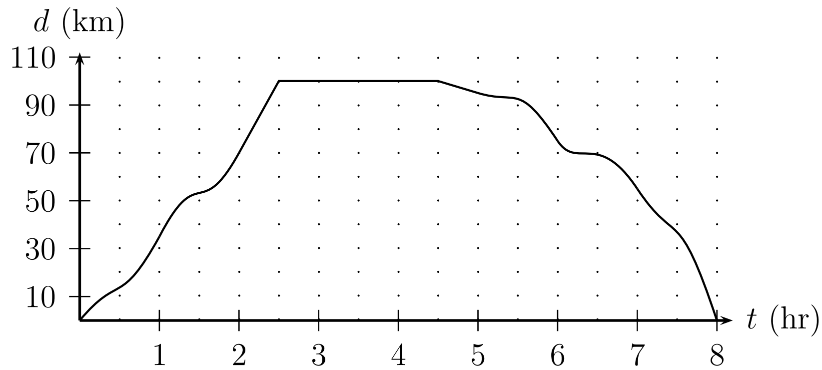
3. Show that it is possible to make a fair five-sided die. Rules: All sides must be flat, and there are no handles (ala a dreidel).

4. (This problem appeared on a Winter, 2014 Math 115 Exam.) The air in a factory is being filtered so that the quantity of a pollutant, P (in mg/liter), is decreasing exponentially. Suppose t is the time in hours since the factory began filtering the air. Also assume 20% of the pollutant is removed in the first five hours.

   (a) What percentage of the pollutant is left after 10 hours?

   (b) How long is it before the pollution is reduced by 50%?

5. (This problem appeared on a Winter, 2014 Math 115 Exam) A ship's captain is making a round trip voyage between two ports. The ship sets sail from Port Jackson at noon, arrives at Port Kembla some time later, waits there for a while, and then returns to Port Jackson. Let $s(t)$ be the ship's distance, in kilometers, from its starting point of Port Jackson, t hours after noon. A graph of $d = s(t)$ is shown below.



(a) How far is Port Kembla from Port Jackson?

(b) How long does the ship wait in Port Kembla?

(c) What is the ship's average speed during the return trip?

(d) Estimate the ship's instantaneous velocity at 1pm.

(e) Sometime after 5pm, there is a time when the ship's instantaneous velocity is 0 km/hr. When does this occur?

6. (This problem appeared on a Winter, 2012 Math 115 exam. Really!) Enjoying breakfast outdoors in a coastal Mediterranean town, Tommy notices a ship that is anchored offshore. The ship is stationed above a reef which lies below the surface of the water, and a series of waves causes its height to oscillate sinusoidally with a period of 6 seconds. When Tommy begins observing, the hull of the ship is at its highest point, 20 feet above the reef. After 1.5 seconds, the hull is 11 feet above the reef.

(a) Write a function $h(t)$ that gives the height of the ship's hull above the reef t seconds after Tommy begins observing.

(b) According to your function, will the hull of the ship hit the reef? Explain.

(c) What proportion of the time is the ship within 4 feet of the reef?